ERANET+: Electromobility+

CACTUS

Models and Methods for the Evaluation and the Optimal Application of Battery Charging and Switching Technologies for Electric Busses

Duration: 01/June/2012 - 31/May/2015

Partners

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Document: Deliverable 3.1 Methods

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Version: 1.0

Date: 31/May/2014

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Chapter 1

Introduction

The global trend towards clean and energy-efficient vehicles is driven by concerns regarding the impacts of fossil-fuel-based road transport on energy security, climate change and public health. Electrification in particular is understood as providing a potential multitude of opportunities for the use of energy from renewable sources and for the reduction of local emissions and green house gas emissions like no other. In 2009, the European Commission presented the Green Cars Initiative aimed at encouraging the development and market uptake of clean and energy-efficient vehicles. This strategy will enable the environmental impact of road transport to be reduced and will boost the competitiveness of the automobile industry.

1.1 Problem

The use of public transport is an environmentally friendly way to travel. If more and more passenger cars will be powered by electrical energy in the future, public transport companies will be forced to convert their diesel busses into electric busses in order not to lose this advantage.

The requirements of busses are different to those of passenger cars. A bus covers an average distance of 250 to 300 km each day. The bus itself has a weight of, for example, 14-17.5 t (Solaris Urbino 18), 28 t (MAN NG 313) or 26.6 t (Mercedes O 405 GN). A suitable battery that would enable the bus to run for such a long distance without having to be recharged would be far too big, heavy and expensive. In order to overcome this problem, several approaches are currently being investigated, for example switching the battery and the short inductive charging of supercapacitors at bus stops. With these technical solutions, which combine vehicles and infrastructure, fully electric busses should be enabled for use in public transportation.

Assumptions:

1. In the near future, there will be no batteries for fully electric busses which provide the daily output of 300 km without needing to be recharged and which would be acceptable in terms of their size, weight and cost.

2. No technical approach that is currently being investigated will be equally suitable for all public transport companies.

3. In any case, investment costs for vehicles, in-vehicle components and infrastructures (e. g. battery charging or battery switching facilities) will be very high for public transport companies.

The following conclusion is drawn: The available technical approaches and solutions must be considered separately against the prerequisites and requirements of every single public transport company in terms of transportation, technical, economic and environmental aspects. Only on this basis can a decision for a technology that optimally meets the requirements of a public transport company be made.

1.2 Objectives

Technical solutions to enable fully electric busses should be evaluated so that they reflect the prerequisites and requirements of the participating public transport companies. The ultimate goal of the project is to find the best technical solution for the participating public transport companies HVB, MVB, PVGS and PKM depending on their real input data (timetable, vehicle operation plan, etc.), which in most cases may mean minimising the investment and operational costs. Of course, the best solution may vary between the participating public transport companies due to the strongly different prerequisites, assignments and aims. The best solution does not only involve a technology, but also its optimal application.

To achieve this aim, models of all relevant transportation, technical, economic and ecological values will be elaborated. Methods will be developed with which the question as to the most suitable technical solution (depending on the input values) can be answered and which help to apply the technical solution found in an optimal way. A software tool will be developed with which the different solutions can be easily compared. It should be possible to study the gradual integration of fully electric busses into existing fleets of diesel, natural gas and hybrid busses.

The preliminary studies with the participating public transport companies will be lead into recommendations for the actors in the field of technology development, namely the manufactures and researchers of fully electric busses and the corresponding infrastructure. The role of the CACTUS project can be seen in Figure 1.1.

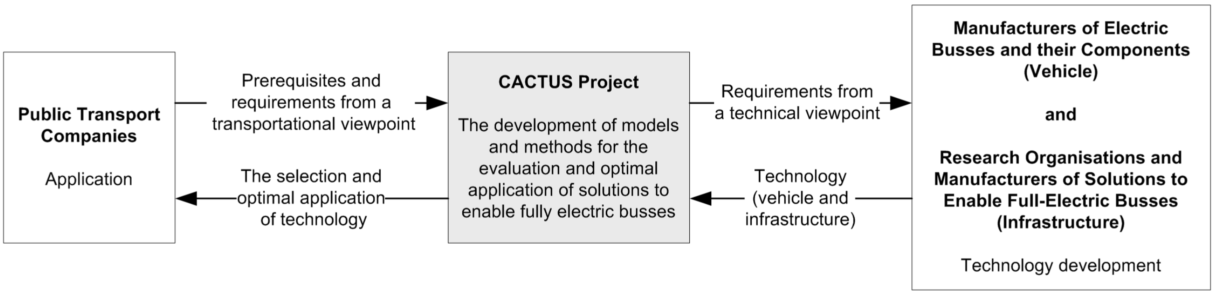


Figure 1.1: The role of the CACTUS project

1.3 Concept

In the CACTUS project, considerations concerning techniques for fully electric busses will be made to decide which best fits a public transport company's needs. This requires a series of detailed questions to be answered. Some general questions are:

- Is it possible to keep to the timetable with a given configuration (all technical and strategic elements requiring the operation of fully electric busses), a given vehicle fleet (including those with mixed engines) and a given vehicle operation plan?
- How high are the investment and operational costs?

In this context, several optimisation issues arise, some of which are listed here:

- What should the operation plan look like so the timetable can be kept to?
- Where the charging or exchanging facilities have to be located?

In the CACTUS project, methods that can be used to answer these will be developed.

Finally all models and methods are used to build a software tool to be used for practical applications. Figure 1.2 shows the general data flow from the input data to the results. Figure 1.3 shows the internal structure of the tool and the two step iterative optimization. Chapter 2 describes the simulation methods and chapter 3 describes the algebraic optimization.

1.4 Scope of this Deliverable

On the basis of collected issues the necessary input variables will be identified and the output variables will be specified. The output variables allow a quantitative assessment and comparability to transportation, technical, economic and ecological aspects. If required input variables are not yet available in electronic form will be collected and prepared, so

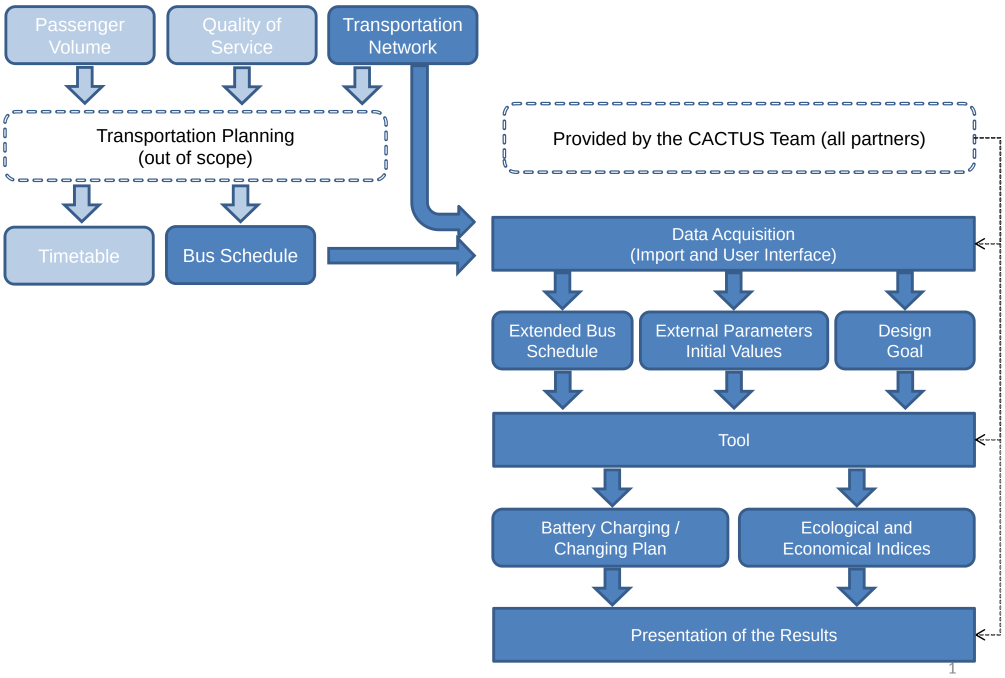


Figure 1.2: Data flow in the tool to be developed

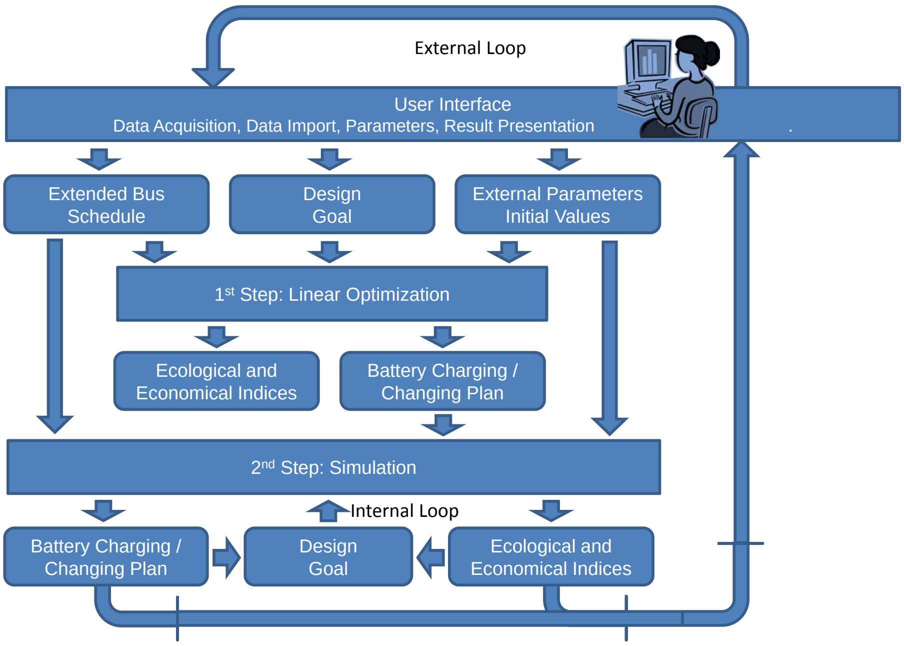


Figure 1.3: Structure and usage of the tool.

that they can be electronically processed. All variables and their relations to each other will be modeled, so the methods to be developed in WP3 can work with. In Deliverable 1.1 the questions have been collected which will be answered within the CACTUS project. The Deliverable 1.1 only mentioned ‘what is the problem’ not ‘how will the problem be solved’ (this is part of Deliverable 2.1 and moreover Deliverable 3.1).

Deliverable 1.2 provides a large collection of current technologies for enabling fully electric buses as well as a broad overview to current available electric buses. The technologies includes lithium-ion batteries, inductive charging, charging via pantograph on the run, exchanging the battery and the application of super capacitors.

The models presented in this Deliverable 2.1 are simplified images of the reality. They are divided into transportation, technical, economic and ecological complexes. Some will be considered as input values and some as output values. Methods to be developed in WP3 works with input values and delivers output values. Models are the basis of the methods to be developed in WP3. The models are a pre-stage of implementation. That means it is aimed to write the models in source code.

Deliverable 3.1 provides several methods in order to answer the questions from Deliverable 1.1. There are simulation and optimization for transportational and technical issues as well as special methods of consideration for economic and ecological issues. Depending on the question one or more of them will be applied.

Chapter 2

Simulation

Simulation is a method for emulating complex systems. This method is chosen within the CACTUS project because here it is neither possible to really investigate electric busses nor combinations of energy storage types (battery, ultracapacitor, flywheel) and energy transmission types (conductive, inductive) which are partially not realised so far. The method of simulation helps to investigate such combinations in order to meet the needs of the associated public transport companies participating in the CACTUS project.

2.1 Aims of Simulation within the CACTUS Project

The simulation method aims at answering the following questions of Deliverable 1.1:

2.1 Energy Consumption

a) How the energy consumption depends on the mass of the whole bus (the bus itself, the battery and the passengers)?

b) How the speed profile influences the energy consumption of the electric bus?

c) How the energy consumption depends on the height profile?

d) How the energy consumption of the electric bus depends on the outside temperature through air conditioning?

e) How much is the influence of the energy recovery (e.g. when braking) on the energy consumption?

2.2 Battery Capacity

a) Is the battery capacity sufficient to a given operation plan?

b) What battery capacity is needed so that the bus never runs into an empty battery fault?

How these questions can be answered by simulation is described in a later section.

2.2 Input Variables

The input variables directly result from the models described in Deliverable 2.1. This includes transportation models like the transport network, the timetable, the road network, the topography as well as technical models like busses, energy storage and energy transmission systems. Detailed relations between the specific processes to be simulated and the models from Deliverable 2.1 are made in the respective subsections.

2.3 Fundamental Approach

Simulation is performed by equidistant time steps. An appropriate time resolution seems to be 1 second. A global timer is the reference value for all processes to be simulated. Every process (e.g. the run of a bus or charging the battery at a charging station) has a starting time or a starting event. When the global timer is equal to the starting time or the starting event occurs, the simulation of the process starts. A process is finished when the global timer reaches a determined time (e.g. planned charging process of at a bus stop) or when a process is completed (e.g. a battery has been fully recharged).

Algorithm 1

```
public interface Process {
    public void performSimulationStep(int time);
}

Process[] p;

for time from 0 to n do
    for i from 0 to (number_of_processes - 1)
        p[i].performSimulationStep(time);
```

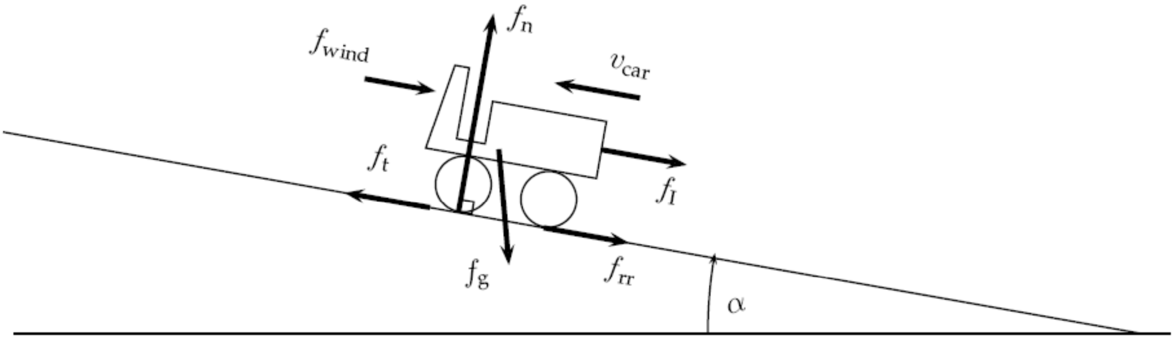


Figure 2.1: Forces acting on a car [18]

Algorithm 1 shows the fundamental approach in pseudo code by the help of the interface mechanism known from the Java programming language. Each process to be simulated has to implement the `Process` interface with the `performSimulationStep` method. The global timer `time` counts from 0 to n and calls every process by handing over the current value of the global timer. Each process has to decide what to do within the elapsed time step. The global timer always increases by equidistant time steps.

2.4 Basic Simulation Processes

There are several basic processes to be simulated: moving the bus including acceleration and deceleration as well as battery charging and battery changing. All of them are described in detail during the following subsections. These basic processes will be used for simulating compound processes later.

2.4.1 Moving

Starting at Figure 2.1 and Equations (2.1) and (2.2) the energy for moving the bus - depending on the speed, the acceleration and the rolling resistance of the tires, the wind, and the angle of the driving surface - can be calculated. The traction force of a vehicle can be described by the following two equations [18]:

$$F_t = \underbrace{M_{car}\dot{v}_{car}}_{f_I} + \underbrace{M_{car} \cdot g}_{f_g} \cdot \sin\alpha + \underbrace{\overbrace{sign(v_{car})M_{car} \cdot g \cdot \cos\alpha}^{f_n} \cdot c_{rr}}_{f_{rr}} + \underbrace{sign(v_{car} + v_{wind})\frac{1}{2}\rho_{air}C_{drag}A_{front}(v_{car} + v_{wind})^2}_{f_{wind}} \tag{2.1}$$

$$c_{rr} = 0.01\left(1 + \frac{3.6}{100}\right)v_{car} \tag{2.2}$$

where

F_t	$[N]$	Traction Force of the vehicle
f_I	$[N]$	Inertial force of the vehicle
f_{rr}	$[N]$	Rolling resistance force of the vehicle
f_g	$[N]$	Gravitational force of the vehicle
f_n	$[N]$	Normal force of the vehicle
f_{wind}	$[N]$	Force due to wind resistance
α	$[rad]$	Angle of the driving surface
M_{car}	$[kg]$	Mass of the vehicle
v_{car}	$[m/s]$	Velocity of the vehicle
$\dot{v}_{car}$	$[m/s^2]$	Acceleration of the vehicle
$g = 9.81$	$[m/s^2]$	Free fall acceleration
$\rho_{air} = 1.2041$	$[kg/m^3]$	Air density of dry air at 20Â°C
c_{rr}	[-]	Tire rolling resistance coefficient
C_{drag}	[-]	Aerodynamic drag coefficient
A_{front}	$[m^2]$	Front area
v_{wind}	$[m/s]$	Headwind speed

The gradient of the way simply results from the elevations of the two successive locations on the run. Equation (2.4) calculates the gradient of the way in degrees, where Δh is the elevation difference between location one and location two and s is the distance between location one and location two.

$$\Delta h = h_{location1} - h_{location2} \tag{2.3}$$

$$\alpha = tan^{-1}\left(\frac{\Delta h}{s}\right) \tag{2.4}$$

This simplest form of consideration assumes a constant gradient on the way between two successive bus stops. But in reality, this is the case only very seldom. Instead, there might be higher and lower gradients from the above calculated gradient on the way. So, this approach ignores these elevation differences between the two bus stops. Nevertheless, of course, the elevation profile can be refined much more, what is a matter of data resolution and data quality and effort for data acquisition.

Then the energy consumption over a period of time is calculated by equation (2.5).

$$E(t) = F_t \cdot \frac{\Delta t}{3600} \tag{2.5}$$

where

E	$[Wh]$	Energy consumption
Δt	$[s]$	Period of time

The unit conversion is done by 1 kWh = 1,000 Wh = 3,600,000 Ws. So, this amount of energy is needed for moving the bus. The mass of the bus in [kg] includes the empty weight of the bus, the weight of the battery and the weight of the passengers. On the chain from the battery to the wheels energy losses occur. The electrical engine itself may have some. The real energy consumption therefore is calculated by equation (2.6).

$$E_{total} = \eta_{Engine} \cdot E_{total} \tag{2.6}$$

In order to apply the equations (2.3) to (2.6) the path is divided into parts with either constant speed or constant acceleration/deceleration. Figure 2.2 sketches a very simplified speed profile of a way between two successive bus stops. Starting at the source bus stop the bus accelerates with constant acceleration until the maximum speed allowed is reached. Then the bus moves with constant speed until braking is required. Finally, the bus decelerates with a constant negative acceleration until it stops at the destination bus stop. Of course, in reality the speed profile is not so linear caused by stops at traffic lights and pedestrian crossings, deceleration before curves and acceleration after curves and so on.

The total energy consumption for moving the bus on a way between two successive bus stops is achieved by equation (2.7). It is the sum of the energy consumption for acceleration, for moving with constants speed and for deceleration.

$$E_{Way} = E_{Acceleration} + E_{ConstantSpeed} + E_{Deceleration} \tag{2.7}$$

The remaining charging level of the energy storage after solving a way is simply calculated by equation (2.8).

$$E_{Battery} = E_{Battery} - E_{Way} \tag{2.8}$$

In order to come to more realistic viewpoint, the real speed profile must be considered which mostly is non-linear. Figure 2.3 depicts in a speed-time-diagram a fictive non-linear

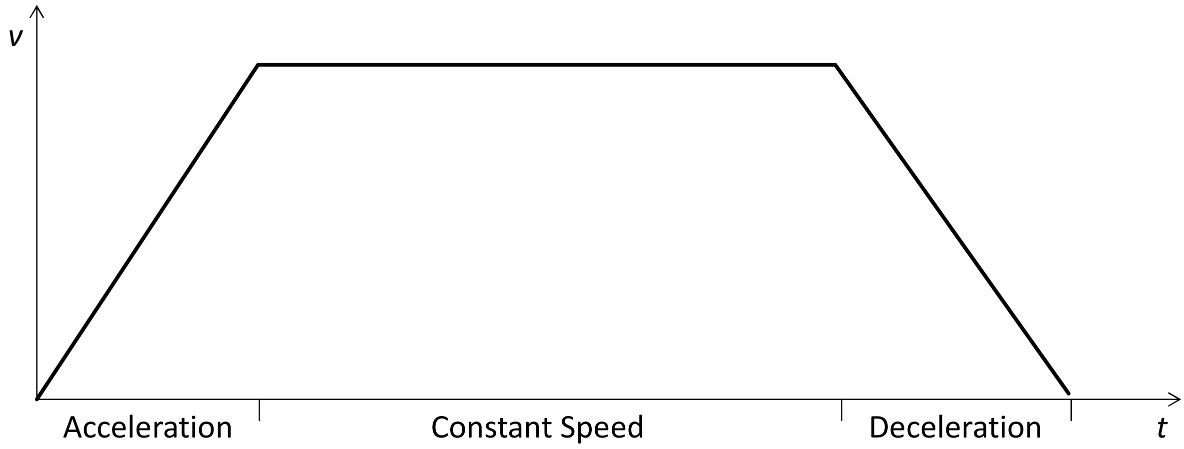


Figure 2.2: Dividing the parts of a way

speed profile of the acceleration phase of a bus. The vertical lines within the diagram mark equal time periods. The default duration of such a time period is one second.

For each time period the speed values of v_t and v_{t+1} are compared. If $v_t = v_{t+1}$ then the speed is constant, if $v_t < v_{t+1}$ then the bus accelerates and if $v_t > v_{t+1}$ then the bus decelerates. Figure 2.4 provides examples of these three types in a v-t-diagram.

Relation to the Models

As described in Deliverable 2.1, the length between two successive bus stops is known from the distance attribute of the `Path` class. The gradient of a way is known from the `ElevationProfile` class (referenced by the `Path` class) which links locations with heights (`ElevationProfileEntry` class with the attributes `location` and `height`).

The speed profile is known from the `SpeedProfile` class (referenced by the `Path` class) which links locations with speeds (`SpeedProfileEntry` class with the attributes `location` and `speed`).

The number of passengers is known from the `Move` class which represents the run between two successive bus stops. The number of passengers is multiplied by the chosen average weight per person (70 kg by default). The resulting value is added to the mass of the bus. The empty mass of the bus is known from the `Bus` class (attribute `empty weight`).

2.4.2 Moving with Constant Speed

When moving with constant speed, neither acceleration nor deceleration takes place. Equation (2.2) therefore can be reduced to equation (2.9).

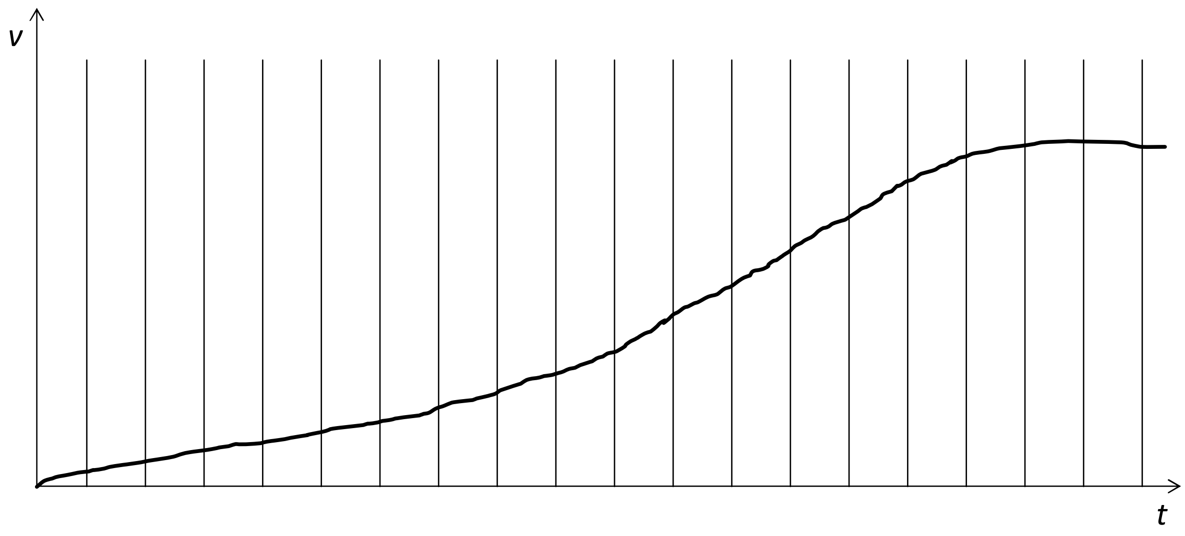


Figure 2.3: Sample acceleration phase divided into several equal periods of time

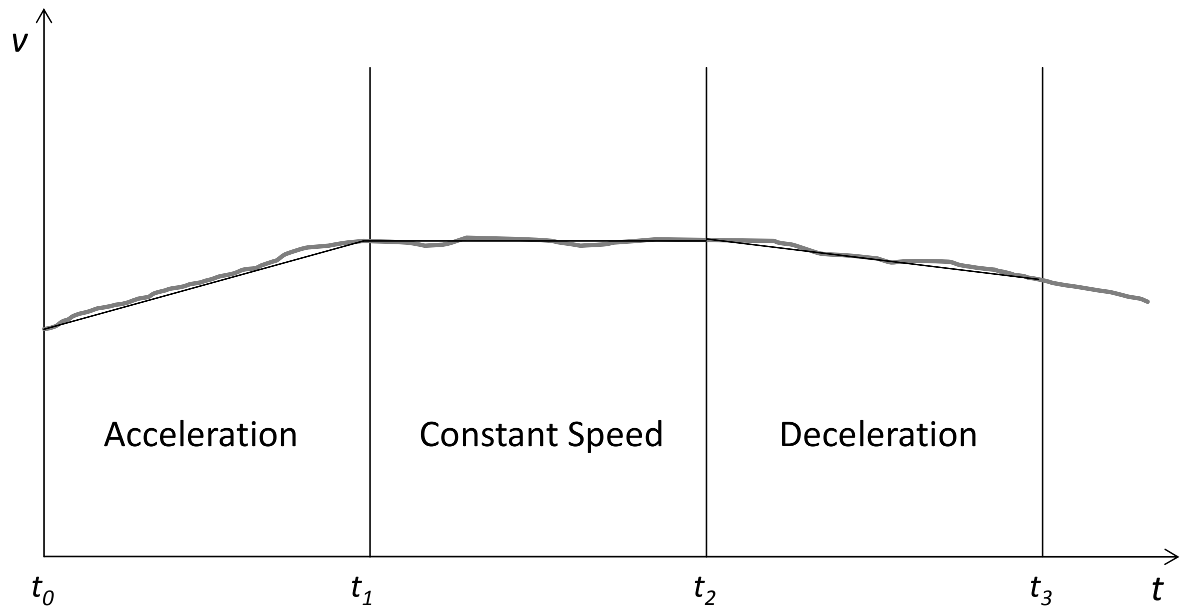


Figure 2.4: Types of moving within a period of time

$$F_t = f_g \cdot \sin\alpha + f_n \cdot c_{rr} + f_{wind} \tag{2.9}$$

where

$$v = min(v_{Bus} \mid v_{Way}) \tag{2.10}$$

The energy consumption for this part of the way is calculated by equation (2.5). By default, the maximum speed of the bus is 50 km/h = 13.89 m/s which is the allowed maximum speed within villages in Germany. It is assumed that a bus is able to reach such a speed value.

2.4.3 Acceleration

When starting at the first bus stop the bus accelerates from a speed of 0 m/s up to the maximum speed which is defined by equation (2.10). By default, the acceleration of the bus is constantly 1 m/s^2 which is, of course, a simplification. The following equation (2.11) derived from equation (2.1) is applied.

$$F_t = M_{car}\dot{v}_{car} + f_g \cdot \sin\alpha + f_n \cdot c_{rr} + f_{wind} \tag{2.11}$$

The acceleration takes time of as described by equation (2.12).

$$\Delta t = \frac{v_{max}}{a} \tag{2.12}$$

Applying equation (2.5) results in the energy consumption in [Wh] for the acceleration part of the way.

2.4.4 Deceleration

When arriving at the second bus stop the bus decelerates from the maximum speed down to 0. The maximum speed again is defined by equation (2.10). By default, the deceleration of the bus is constantly 1 m/s^2 (or a negative acceleration of -1 m/s^2). The following equation (2.13) derived from equation (2.1) is applied.

$$F_t = f_I + f_g \cdot \sin\alpha + f_n \cdot c_{rr} + f_{wind} \tag{2.13}$$

The deceleration takes time of as described in equation (2.14):

$$\Delta t = \frac{v_{max}}{|a|} \tag{2.14}$$

Applying equation (2.5) results in the energy consumption in [Wh] for the deceleration part of the way.

2.4.5 Energy Recovery while Braking

When the bus decelerates, the waste energy can be recovered and stored back in the energy storage. The amount of energy to be recovered depends on the efficiency of the energy recovery unit. By the following equation, the amount of energy which is restored in the energy storage is calculated.

$$E(t) = \eta \cdot P \cdot \Delta t \tag{2.15}$$

By default, the efficiency of the energy recovery is 99%. When no energy recovery is possible, the efficiency has to be set to 0.

Relation to the Models

As described in Deliverable 2.1, the `Regenerative Brake` class which is referenced by the `Electric Bus` class.

2.4.6 Engine

Each engine (it does not matter whether electrical engine or diesel engine) has energy losses and therefore an efficiency. The energy consumption really needed is higher than the energy calculated in equation (2.7). Therefore it is calculated by the following equation (2.16) which considers the efficiency of the engine.

$$E_{total} = \frac{E_{Way}}{\eta_{Engine}} \tag{2.16}$$

Furthermore, the engine has a voltage (unit V) and a power (unit kW). These values will be used to ensure the compatibility of energy sources with energy storages. This help to avoid errors at the planning stage.

2.4.7 Charging

Charging can be done while the bus is standing or while it is moving. We consider that the energy storage is being recharged parallel to consuming energy.

$$E(t) = \eta \cdot P \cdot \Delta t \tag{2.17}$$

with

E	[Ws]	Energy within the energy storage
η	[%]	Efficiency of energy transmission
P	[W]	Charging power
Δt	[s]	Energy transmission period of time

After charging, the charging level of the energy storage of the bus has increased by E. For example, charging with a power of 200 kW for a time period of 30 s results in 200 kWs (200 kW * 30 s) by which the charging level rises. For simplification, a linear increase of the charging level of the energy storage is assumed. In reality this is not the case. The charging curve of the different energy storage types has to be considered later.

There are further properties of the charging process. First of all every energy storage has a maximum charging level which is at 100% or at a specific amount of energy in kWh (capacity). Continue charging keeps the charging level at 100%.

Particularly batteries have a maximum allowed charging power Pmax and an optimum charging power Popt. A charging power higher than the maximal allowed charging power would destroy or irreversible damage the battery. A charging power which is different from the optimum charging power leads to a lower battery lifetime which is relevant for the ecological impact of electric busses.

When charging is performed beside the actual energy transmission process there is time needed for preparing the energy transmission (e.g. connecting to the charging station; someone has to plug in the cable into the socket) and post processing the energy transmission (e.g. disconnecting from the charging station; someone has the plug out the cable from the socket and to stow away it in the bus. Therefore the total period of time for the charging process is calculated by the following equation:

$$t_{total} = t_{connect} + t_{charging} + t_{disconnect} \tag{2.18}$$

All of this has to be considered during the planning phase.

2.4.8 Energy Storage Exchanging

Compared to charging energy storage the process of energy storage exchanging (only possible for batteries, not for ultracapacitors and flywheels) is performed in a very short period of time:

$$t_{total} = t_{exchanging} \tag{2.19}$$

The (fully) recharged battery taken out of the exchanging station now can be used within the bus. The simulation must manage both the empty battery and the recharged battery in order to avoid errors like considering an exchanging station as an infinite source of fully recharged batteries. This means, recharging the empty battery within the exchanging station is simulated as well.

2.4.9 Passengers

The number of passengers influences the total mass of the bus and therefore the energy consumption. The average weight of a person is assumed at 70 kg by default.

In order not to overload the bus the number of seats $n_{seating}$ and the number of standing places $n_{standing}$ must be regarded.

When simulation is performed the average occupancy rate of the bus can be calculated. If the bus is highly occupied a warning is thrown. If the bus is over occupied a critical error is thrown. Passengers waiting at bus stops which eventually cannot be picked up are not simulated.

Relation to the Models

The number of seats and the number of standing places are known from the `Bus` class (attributes with the same name).

2.4.10 Inventory

The inventory comprises all parts of the bus which consume electrical energy excepting the engine. These parts belong to a conventional diesel bus as well but in a diesel bus the electrical energy needed is generated from the combustion engine. There are a lot of inventory devices within a bus in public transportation including air-conditioning, lighting, displays, ticket automat etc. to mention only the most important. Air-conditioning includes heating in winter and cooling in summer. It has the most energy consumption. We assume a continuous energy consumption of any device. That means we consider them as always on. The energy consumption can be calculated by the following equation (2.20):

$$E(t) = P_{Inventory} \cdot \Delta t \tag{2.20}$$

with

E	[Ws]	Energy consumption
$P_{Inventory}$	[W]	Power of the inventory
Δt	[s]	Working duration of the inventory

For example, the air condition may have a power of 900 W. For example, the lights may have a power of 500 W. The energy consumption of the inventory must be added to the energy consumption for moving the bus.

Relation to the Models

The energy consumption of the inventory devices is known from the `Electrical Consumer` class which is referenced by the `Bus` class.

2.4.11 Charging Level

When the bus is simulated the charging level of the energy storage has to be watched. Particularly the lifetime of batteries is strongly decreased when they are totally discharged what results in a negative economic and ecological impact. Usually, the threshold of charging level is at 30% at batteries (minimum charging level). This is regarded as follows:

If the charging level reaches the defined minimum charging level, a warning is thrown. Moving the bus can be optionally stopped at the minimum charging level. If the charging level reaches zero, a critical error is thrown. In this case, the simulation has failed.

2.5 Compound Simulation Processes

This section shows how basic simulation processes are used to build compound simulation processes. Compound simulation processes are understood as simulation activities on a higher level particularly thought for the concerns of the public transport companies.

2.5.1 Run

Remember, a run is a ride of a bus on a bus route from a start bus stop via a set of intermediate bus stops to the end bus stop. Each run is exactly defined in the timetable. A move is understood as a ride of the bus from one bus stop to the successive bus stop without intermediate stops at any other bus stop. The following Algorithm 2 illustrates the basic simulation vehicles compound for simulating a run:

Algorithm 2

```
for each Move of Run do
    for time = 0, t++
        calculate energy consumption for the
        last period of time
    wait for passengers to get on and off the
    bus for a fixed time period
```

We assume a speed profile given for each move which describes the speed of the bus at different points of time. The temporal distance between two adjacent points of time is always equal. By default it is 1 second. For each period of time the energy consumption for moving the bus within this period of time is calculated by equation (2.4) and derivations respective. When simulation is performed the bus stops at each bus stop on the route. It waits at each bus stop for passengers to get on and off for a fixed period of time.

Relation to the Models

A `Run` is an ordered list of objects of the `Move` class. A `Move` object represents the movement of a bus from a bus stop to the successive bus stop. Each `Move` object references a `Path` object which connects two `Stop` objects. Each `Path` object references a `Way Profile` object which brings together `Speed Profile` and `Elevation Profile` values. The `Run` is referenced by the `Timetable`. A `Run` is performed by a `Bus`.

2.5.2 Sequence

A sequence is not only a single run but an ordered list of activities which also includes moves from and to the depot or another parking place (transition move). This is more realistic than simulating simply a single run. The following Algorithm 3 illustrates the course of simulating a sequence whereby activities can be executed at the same time (e.g. moving and charging).

Algorithm 3

```
for time from 0 to n do
    switch Activity of Sequence
        case Standing
        case Charging
            simulate charging
        case Transition Move
            simulate Move
        case Run
            simulate Run
```

Relation to the Models

A `Sequence` contains at least one `Run`. Furthermore a `Sequence` includes moves (class `Move`) from or to the depot and parking places as well as standing times.

2.5.3 Timetable

Simulating a complete timetable or a part of it additionally requires the Operation Plan. Simulating a whole timetable brings together results from all sequences and so delivers values for all considered bus routes. Compared to simulating a single run or a sequence, simulating the timetable helps to avoid errors such as a charging station is occupied by more busses than allowed. Simulating a timetable requires time parallel simulation of several busses.

Algorithm 4

```
for time from 0 to n do
    for i from 0 to (number_of_sequences - 1)
        sequence[i].performSimulationStep(time);
```

Relation to the Models

A `Timetable` contains all `Run` objects. The `Operation Plan` contains all `Sequence` objects.

2.6 Output Variables

The main output value of the simulation is the energy consumption (for all considered bus routes) depending on the total mass of the bus, the speed profile and the elevation profile. Changes of the mass, the speed profile or the elevation profile leads to different energy consumption. The energy consumption is an important input value for the optimization method as well as for the economical and ecologic impact, which is shown later in this deliverable (Chapters 3 to 5).

Further output values are the profile of the battery charging level, the number of charging cycles of the energy storages and the occupation times of charging tracks and charging and exchanging stations. Here, simulation ensures that no such track or station is used beyond its capacity.

By interpreting these output values over several simulation runs with variations of parameters, simulation is also used to answer more general questions as Deliverable 1.1 tells. How this can be done is explained in within next section.

2.7 Answering the Questions from Deliverable 1.1

This section explains in detail how the questions from Deliverable 1.1 can be answered by simulation. Particularly the compound simulation activities will be applied. The numbers in parentheses in the headline of the following subsections relate to the number in Deliverable 1.1.

For all methods described in the following sections the charging and exchanging infrastructure is given and is not changed during the simulation process or between several simulation runs. In order to optimize the charging and exchanging infrastructure a method other than simulation is applied (details in the next chapter of this deliverable).

2.7.1 How the energy consumption depends on the mass of the whole bus (the bus itself, the battery and the passengers) (2.1a)?

The relation between the mass and the energy consumption is given by equation (**??**). In order to see how this is in the context of a concrete public transport company, several simulation runs have to be performed while the mass of the bus is varied. All other input parameters like sequence, bus, speed profile, elevation profile and outside temperature are constant. The input parameters energy recovery and battery capacity do not influence the energy consumption and therefore are not relevant. Algorithm 5 outlines the outer loop of the simulation runs.

Algorithm 5

```
for i from 1 to n do
    E[Mass] := Simulate(Bus, Sequence, Mass[i], SpeedProfile,
               Temperature, ElevationProfile)
```

$E[Mass]$ can be depicted in a diagram. By this, the influence of the mass on the energy consumption can be seen. The minimum of E can be easily read. The correlation between the mass of the bus and the energy consumption might result in a function (linear or exponential).

2.7.2 How the speed profile influences the energy consumption of the electric bus (2.1b)?

Speed and acceleration are major cause variables of the energy consumption as given by equation (2.1). In order to see how the speed profile influences the energy consumption of the electric bus, several simulation runs have to be performed while the mass of the bus is varied. All other input parameters like sequence, bus, mass, elevation profile and outside temperature are constant. The input parameters energy recovery and battery capacity do not influence the energy consumption and therefore are not relevant. Algorithm 6 outlines the outer loop of the simulation runs.

Algorithm 6

```
for i from 1 to n do
    E[i] := Simulate(Bus, Sequence, Mass, SpeedProfile[i],
            Temperature, ElevationProfile)
```

$E[i]$ can be depicted in a diagram where the x-axis represents the speed profile and the y-axis represents E. By this, the influence of the speed profile on the energy consumption can be seen. The minimum of E can be easily read.

Of course, the speed profile is not a physical quantity like mass, time or length. But it can be described by several variables: total changes of speed, number of acceleration operations, number of deceleration operations, average/minimum/maximum acceleration and deceleration values and number of total stops. These variables are used to define, generate or record different speed profiles.

2.7.3 How the energy consumption depends on the height profile (2.1c)?

However, the height profile (or elevation profile) cannot be changed by human behavior like speed profile. Nevertheless, it seems to be interesting to find out the relation between different elevation profiles and the total energy consumption. In order to find out this relation for a public transport company, several simulation runs with different elevation profiles have to be executed. The following Algorithm 7 outlines the outer loop of the simulation runs.

Algorithm 7

```
for i from 1 to n do
    E[i] := Simulate(Bus, Sequence, Mass, SpeedProfile,
            Temperature, ElevationProfile[i], EnergyRecovery)
```

$E[i]$ can be depicted in a diagram where the x-axis represents the elevation profile and the y-axis represents E. By this, the influence of the elevation profile on the energy consumption can be seen.

Of course, the lower the total differences of elevation meters and the lower the gradient the lower is the energy consumption. Of course, this relation is obviously. But it is interesting to know the total amount of energy consumption depending on the elevation profile. Like the speed profile, the elevation profile is not a physical quantity like time or mass. But it can be described by some variables like total differences of elevation meters and average/minimum/maximum gradient.

2.7.4 How the energy consumption of the electric bus depends on the outside temperature through air conditioning (2.1d)?

The relation between the outside temperature and the energy consumption is largely determined by the energy consumption of the inventory. In particular, the energy consumption of air conditioning is high in winter when heating is needed and in summer when cooling is needed. A high energy consumption of the inventory influences the range of the bus and therefore may cause more or longer charging or exchanging operations.

In order to see how this is in the context of a concrete public transport company, several simulation runs have to be performed while the temperature is varied. All other input parameters like sequence, bus, mass, speed profile and elevation profile are constant. The input parameters energy recovery and battery capacity do not influence the energy consumption and therefore are not relevant. Algorithm 8 outlines the outer loop of the simulation runs.

Algorithm 8

```
for i from 1 to n do
    E[i] := Simulate(Bus, Sequence, Mass, SpeedProfile,
            Temperature[i], ElevationProfile, EnergyRecovery,
            BatteryCapacity)
```

$E[i]$ can be depicted in a diagram where the x-axis represents the outside temperature and the y-axis represents E. By this, the influence of the outside temperature on the energy consumption can be seen. The correlation between the mass of the bus and the energy consumption might result in a function (linear or exponential).

2.7.5 How much is the influence of the energy recovery (e.g. when braking) on the energy consumption (2.1e)?

This question can be answered by simulating once with energy recovery and simulation once without energy recovery or with energy recovery at different efficiencies. This leads to differences in the minimum battery charging level. By changing the efficiency (this could be done automatically by the simulation environment), the differences can be visualized in a diagram where the x-axis represents the efficiency pf energy recovery and the y-axis represents the energy consumption. Algorithm 9 outlines the outer loop of the simulation runs.

Algorithm 9

```
for i from 1 to n do
    E[i] := Simulate(Bus, Sequence, Mass, SpeedProfile,
            Temperature, ElevationProfile, EnergyRecovery[i],
            BatteryCapacity)
```

2.7.6 Is the battery capacity sufficient to a given operation plan (2.2a)?

The electric busses are simulated with the given operation plan (represented by `Sequence`). The question can be answered with yes if no battery empty fault warning or error is thrown (this depends on the specification of the user, that is the operating public transport company). If the simulation runs into an empty battery fault error the battery capacity definitively is too low. However, changing the operation plan (e.g. increasing the frequency of charging or exchanging operations) may help to solve the problem too.

2.7.7 What battery capacity is needed so that the bus never runs into an empty battery fault (2.2b)?

Like in the precedent question 2.2a during the simulation the charging level of a battery is decreased though consuming energy and it is increased by charging operations. Thus, the charging level changes over the time during the simulation run. The minimum charging level here is defined as the lowest charging level the battery has during the simulation. Figure 2.5 provides a fictive sample of this.

Now to answer the question 2.2b, the simulation has to be repeated several times using a different battery capacity each run. At each run, the charging level of the battery will lead to another minimum. There is a direct relation between the capacity and the minimum charging level: A lower capacity leads to a lower minimum charging level. The minimum (and maybe optimum value with respect to economic aspects) value of the battery capacity can be found out by taking the minimum charging level recommended by the manufacturer. Algorithm 10 outlines the outer loop of the simulation runs.

$ChargingLevel[i]$ can be depicted in a diagram where the x-axis represents the battery capacity and the y-axis represents the minimum charging level. By this, the influence of the capacity on the minimum battery charging level can be seen. More frequent charging operations may lead to a higher minimum charging level of the battery.

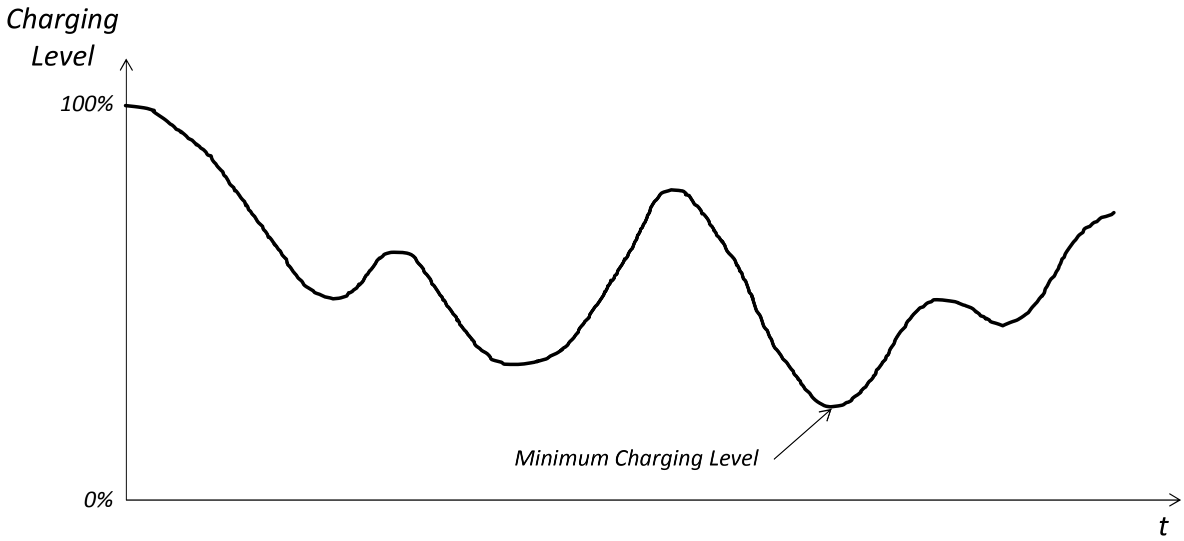


Figure 2.5: Charging level of an energy storage over time

Algorithm 10

```
for i from 1 to n do
    ChargingLevel[i] := Simulate(Bus, Sequence, Mass,
                        SpeedProfile, Temperature,
                        ElevationProfile, EnergyRecovery,
                        BatteryCapacity[i])
```

Chapter 3

Optimization

The aim of this work is to get first results in a short run time. For this reason we use linear models and average values for input variables. This approach is at the expense of accuracy. However, the results give some hints and good start values for an ex post simulation to be run for further improvements.

We consider tree kinds of charging facilities: Charging on the road via inductive or conductive energy transfer (*way charging*), charging at bus stops *point charging* and changing batteries (*swapping*) at swapping stations, typically at especially equipped bus stops. The problem is to determine the location, length and power for way charging respectively the location and energy amount for charging stations and for swapping stations.

3.1 A Linear Model

3.1.1 Basic technical Model

The model allows to calculate the current energy for each bus after running a cycle without charging recursively. The start value $\mathbf{e_0}$ is the initial energy and the model parameters are the lower bound $\mathbf{e_{min}}$ and upper bound $\mathbf{e_{max}}$ for the battery charges.

$$\mathbf{e}\big((k+1)\,T\big) = \mathbf{e}\big(kT\big) - \mathbf{SW}\cdot\mathbf{e}^{-} \qquad \mathbf{e_{min}} \le \mathbf{e}\big(kT\big) \le \mathbf{e_{max}} \qquad k \in \mathbb{N}_0 \tag{3.1}$$

$$\mathbf{e}(0) = \mathbf{e_0} \qquad \mathbf{e_{min}} \le \mathbf{e_0} \le \mathbf{e_{max}} \tag{3.2}$$

Each bus $i \in \mathbb{B}$ has to be equipped with a battery of sufficient capacity and enough initial energy to run at least one cycle of duration of t_{ci}.

Batteries are modeled as a linear storage with a certain efficiency η for charging. Even the system is modeled linear there are some boundaries which must be taken into account. The battery's capacity e_{max} which is the absolute limit for the battery charge cannot be overdrawn. In order to achieve a long battery lifetime it should be run in a range between

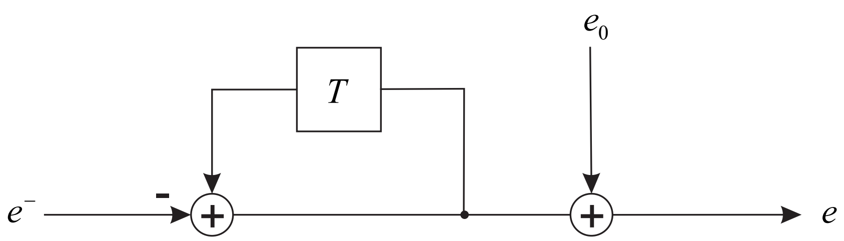


Figure 3.1: Simple energy model for cylic operation with cycle time T

the lower bound e_{min} and the upper bound e_{max} The charging power[1] is limited and we use a maximal charging power p_{max} which can be chosen as a parameter.

3.1.2 Optimization

Linear optimization is a powerful method for addressing a wide range of applied optimization problems. Typical application areas are resource allocation, production scheduling, warehousing layout, transportation scheduling, facility location, portfolio optimization, parameter estimation and many more. A mathematical optimization problem minimizes or maximizes a function value with respect to some constraints (see [19]).

The optimization problem over $\mathbb{R}^n$ is given by the linear objective function

$$c_1x_1 + c_2x_2 + \cdots + c_nx_n \qquad c_i \in \mathbb{R}$$

whose value is to minimize or to maximize and by m constraints

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n \left\{ \begin{array}{c} \leq \\ = \\ \geq \end{array} \right\} b \qquad a_i \in \mathbb{R}$$

The canonical form for the linear optimization is given by a maximum problem with some inequations.

$$\mathbf{c}^T \cdot \mathbf{x} \rightarrow \max \qquad \text{objective function} \qquad \mathbf{c} \in \mathbb{R}^n, \mathbf{x} \in \mathbb{R}^n \tag{3.3}$$

$$\mathbf{A} \cdot \mathbf{x} \leq \mathbf{b} \qquad \text{constraints} \qquad \mathbf{A} \in \mathbb{R}^{m \times n}, \mathbf{b} \in \mathbb{R}^m \tag{3.4}$$

Linear optimization is used to minimize the installation effort for the stationary facilities. For this reason we use a minimum form [3.5] and greater equal instead of less equal inequations [3.6]

[1] The introduced model uses the charging power on behalf of the charging current. So we do not need to know the voltage of the battery.

$$\mathbf{c}^T \cdot \mathbf{x} \rightarrow \min \qquad \text{objective function} \tag{3.5}$$

$$\mathbf{A} \cdot \mathbf{x} \geq \mathbf{b} \qquad \text{constraints} \tag{3.6}$$

Depending of the application some variables x_i must be integer which leads to a MILP (Mixed Integer Linear Programming) model.

In the following sections we develop some optimization models $(\mathbf{c}, \mathbf{A}, \mathbf{b})$ for different policies of operation.

3.2 Network and Schedules

We introduce the formal definition of the transportation network model and the bus schedules.

3.2.1 The transportation network

The transportation network consists of *bus stops* $\mathbb{S}$ and *ways* $\mathbb{W}$. A way is a sequence of roads connecting the two bus stops. The corresponding mathematical model is a directed graph $N = (\mathbb{S}, \mathbb{W})$. The nodes $\mathbb{S}$ represent the bus stops and the edges $\mathbb{W}$ represent the ways connecting two bus stops respectively. Each way is attributed to average values for drive time t_D, length len and energy consumption e^-. The structure of the net can be expressed by the incidence matrix $\mathbf{B}$.

$$\mathbf{B} = \{-1, 0, 1\}^{|\mathbb{S}| \times |\mathbb{W}|} \qquad b_{i,j} = \begin{cases} -1 & \text{if a way j starts at bus stop i} \\ 1 & \text{if a way j ends at bus stop i} \\ 0 & \text{otherwise} \end{cases} \tag{3.7}$$

In most cases it is not necessary to build the network N explicitly. Vectors $\mathbf{t_D}$, $\mathbf{len}$ and $\mathbf{e}^-$ over all ways are sufficient. The required parts of the net are represented by ways connecting bus stop and sequences of ways which are used by bus lines. This is part of the *bus schedule* (see below).

$$\text{Buses} \qquad \mathbb{B} = \{1, 2, \ldots i, \ldots l\} \tag{3.8}$$

$$\text{Ways} \qquad \mathbb{W} = \{1, 2, \ldots j, \ldots m\} \tag{3.9}$$

$$\text{Bus stops} \qquad \mathbb{S} = \{1, 2, \ldots k, \ldots n\} \tag{3.10}$$

3.2.2 Timetables and bus schedules

While *timetables* are the view of the passengers, *bus schedules* are the perspective of the bus drivers. In both cases departure times are given but there are no arrival times specified. The travel time t_d between two bus stops depends on the current traffic situation. This delays which accumulate and cannot be avoided. To keep this effect under control there is a *slack time* t_{slack} between the end of cycle k and the begin of cycle $k+1$. A lower bound for t_d is given by driving at maximal allowed speed and green traffic lights at any time.

In urban areas the buses serve their lines in a cyclic mode. This is essential to the introduced model because any time domain details become obsolete and the data structures and the computation time can be kept much smaller. For each bus there is a schedule vector sw over all ways $j \in \mathbb{W}$. sw_j is the frequency of the bus on the way $j \in \mathbb{W}$ for a global cycle of time T. The global cycle time is a period of time in which the cycle times of all buses fit. This is the least common multiple (lcm) of all bus cycle times t_c.

$$global\ cycle\ time \qquad T = \operatorname*{lcm}_{i \in \mathbb{B}}(t_{ci} + t_{slacki}) \tag{3.11}$$

The way based schedule matrix $\mathbf{SW}$ consists of one row for schedule vector of each bus. This model is sufficient for the problem of way charging. For battery swapping at bus stops the way based version $\mathbf{SW}$ of the schedule matrix can be converted to a bus stop based version SS of the schedule matrix by multiplying $\mathbf{SW}$ with the positive part of the network incidence matrix $\mathbf{B}$.

$$\mathbf{SW} \cdot abs(\mathbf{B}^T)/2 = \mathbf{SS}$$

$$\mathbf{SW} = \begin{pmatrix} sw_{11} & \cdots & sw_{1m} \\ \vdots & \ddots & \vdots \\ sw_{l1} & \cdots & sw_{lm} \end{pmatrix}$$

sw_{ij} : Frequency of driving bus i over way j

$$\mathbf{SS} = \begin{pmatrix} ss_{11} & \cdots & ss_{1n} \\ \vdots & \ddots & \vdots \\ ss_{l1} & \cdots & ss_{ln} \end{pmatrix}$$

ss_{ik} : Frequency of stopping bus i at bus stop k

3.3 Charging on ways

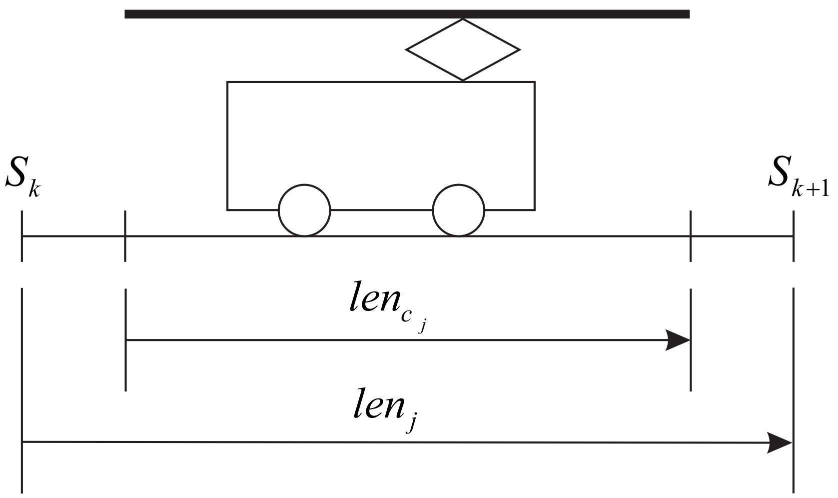
This policy is is robust against any delay and the basic principle is well known from trolley buses. The stationary installations are needed only for some selected parts of the transportation network. Nowadays different techniques based on inductive (e.g. []) or conductive (e.g. []) power transfer are available. Tailored solutions can be integrated in big cities as well as in historic sections of old towns.

From the technical view it is not only the charging aspect but the saving of battery energy while power transfer is active. The precondition is the availability of enough power for charging and driving.

3.3.1 Way charging model

For every way the attributes length len, energy consumption e^-, potential energy for recovering r^+ and average drive time t_d can be acquired by analysis of time tables and height profiles or by measuring. For charging on ways we define a *power rail* of the length l_c and a charge factor $x = len_c/len$. This power rail is topic for any characteristic like contact wire or inductive coils. See table 3.3.1 for definitions and ranges.

Attribute	Range	Description	
len	$len > 0$	Length of the way	given by the transportation network
e^-	$e^- \geq 0$	Energy consumption	
r^+	$r^+ \geq 0$	Potential energy for recovering	
t_d	t_d	Drive time	
len_c	$0 \leq len_c \leq l$	Length of the power rail	to be identified
x	$0 \leq x \leq 1$	Charge factor len_c/len	



$$x_j = \frac{len_{cj}}{len_j} \tag{3.12}$$

Figure 3.2: A way j of length len connecting bus stop k and bus stop $k+1$ is partly equipped with a power rail of length len_c

Ways are the logical representation of a sequence of bus stop connections and they can share roads on which the buses drive. Since the basic idea of planning power rails is the multiple use of them by different buses. For this reason additional *way points* might be necessary and they can be treated as unused bus stops in the way charging model. Only ways are used in the way based schedule matrix $\mathbf{SW}$ and extra bus stops representing only way points do not influence the model.

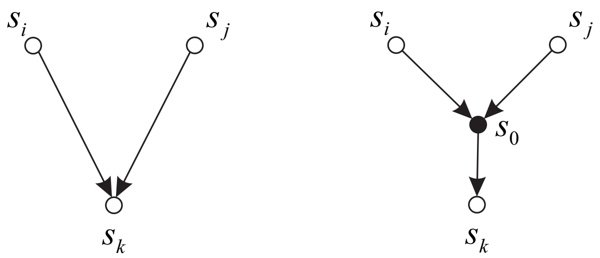


Figure 3.3: Way point s_0 makes a way partly common.

The maximal energy e_i^+ which a bus i can take in a global cycle T is

$$e_i{}^+ = p_{max i} \cdot \sum_{j \in \mathbb{W}} \underbrace{sw_{ij} \cdot t_{dj} \cdot x_j}_{\substack{\text{energy charge for} \\ \text{bus i on way j}}} \tag{3.13}$$

and in general for all buses

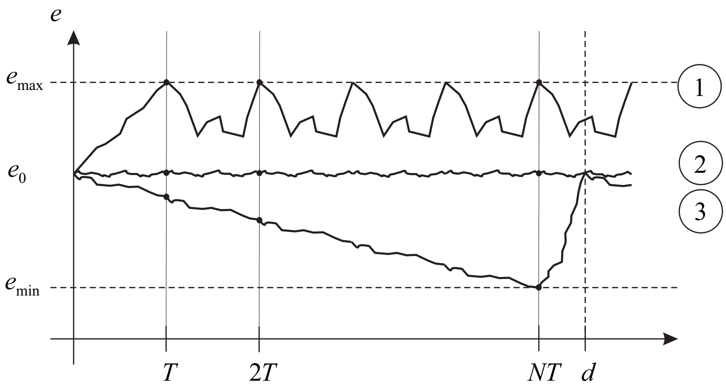
$$\mathbf{e}^+ = \underbrace{diag(\mathbf{p_{max}}) \cdot \mathbf{SW} \cdot diag(\mathbf{t_d})}_{\mathbf{A}} \cdot \mathbf{x} \tag{3.14}$$

$$a_{ij} = p_{max i} \cdot sw_{ij} \cdot t_{dj} \tag{3.15}$$

$$\mathbf{A} = \begin{pmatrix} p_{max1} \cdot sw_{11} \cdot t_{d1} & p_{max1} \cdot sw_{12} \cdot t_{d2} & \cdots \\ p_{max2} \cdot sw_{21} \cdot t_{d1} & p_{max2} \cdot sw_{22} \cdot t_{d2} & \cdots \\ \vdots & \vdots & \vdots \end{pmatrix} \tag{3.16}$$

For charging some ways must be equipped with power rails so that at least each bus can get its required energy by using the maximal allowed charging power p_{max}. At the end of any global cycle the energy e_{target} must be available. According to figure 3.4 three cases for e_{target} can be distinguished.

1. e_{max} must be achieved at the end of any global cycle.
2. e_0 must be kept after any global cycle.
3. e_{min} can be reached after N global cycles at the end of the day with an overnight charge at the bus depot.



$$e_{target} = \begin{cases} e_{max} \\ e_0 \\ e_0 - \frac{e_0 - e_{min}}{N} \end{cases}$$

Figure 3.4: Modes of charge for cyclic operation with n cycles of time T within a day d

The charging infrastructure must offer at least e_{target} for any bus. The charging controller finally prevents an over charging.

$$\mathbf{e_0} - \mathbf{SW} \cdot \mathbf{e}^- + \mathbf{e}^+ \geq \mathbf{e_{target}} \tag{3.17}$$

$$\mathbf{e_0} - \mathbf{SW} \cdot \mathbf{e}^- + \underbrace{diag(\mathbf{p_{max}}) \cdot \mathbf{SW} \cdot diag(\mathbf{t_d}) \cdot \mathbf{x}}_{\mathbf{e}^+ \text{for way charging as defined in (3.14)}} \geq \mathbf{e_{target}} \tag{3.18}$$

3.3.2 Technical view on way charging optimization

The technical constraints are given by (3.18) where $x_i \in \mathbb{R}_+$ must be kept in the range $[0 \ldots 1]$. Equation (3.19) is the form used for optimization. The lower boundary vector $\mathbf{b}$ depends on the target energy e_{target} given by the design goal.

The objective is a minimum overall length of the installed power rail. Finally we get the optimization problem in standard form and in terms of the way charging model:

$$\overbrace{\underbrace{diag(\mathbf{p_{max}}) \cdot \mathbf{SW} \cdot diag(\mathbf{t_d})}_{\mathbf{A}} \cdot \mathbf{x}}^{\text{offer}} \geq \overbrace{\underbrace{\mathbf{SW} \cdot \mathbf{e}^- + \mathbf{e_{target}} - \mathbf{e_0}}_{\mathbf{b}}}^{\text{demand}} \qquad \begin{cases} x_i \in \mathbb{R}_+ \\ 0 \leq x_i \leq 1 \end{cases} \tag{3.19}$$

$$\mathbf{len}^T \cdot \mathbf{x} \to \min \tag{3.20}$$

In many cases it is not possible to install charging facilities on some specific roads. This can be achieved easily by increasing the corresponding coefficients of the objective function. The solution $\mathbf{x}$ gives the relative length and $diag(\mathbf{l}) \cdot \mathbf{x}$ gives the absolute length of the power rail for all ways. The value of the objective function gives the whole length of all power rails.

The optimization checks only the lower bound e_{target} and is based on the maximal charging power for each bus. In a second step the really needed power can be calculated so that the upper bound e_{max} will not be overdrawn and finally the target energy will be reached. The constraints (see equation (**??**)) are changed to be equality and the required mean value of the charging power p can be calculated.

$$diag(\mathbf{p}) \cdot \mathbf{SW} \cdot diag(\mathbf{t_d}) \cdot \mathbf{x} = \mathbf{SW} \cdot \mathbf{e}^{-} + \mathbf{e_{target}} - \mathbf{e_0} \tag{3.21}$$

$$p_i = \frac{\sum_{j=1}^{|\mathbb{W}|} sw_{ij} \cdot e_j^{-} + e_{target_i} - e_{0i}}{\sum_{j=1}^{|\mathbb{W}|} sw_{ij} \cdot t_{dj} \cdot x_j} \tag{3.22}$$

This condition can be realized only by the charging controller on the bus which should charge with an adapted amount of power p.

3.3.3 Economical view on way charging optimization

The technical view results in a set of power rails each specified by its location and length. There might be several optimal solutions which all have the same total length but they could differ in the number of power rails. From the economical view the total number of power rails should be minimized as well. The technical view from the previous section $\mathbf{A}, \mathbf{x}, \mathbf{b}$ become now $\mathbf{A_1}, \mathbf{x_1}, \mathbf{b_2}$. The relative length $\mathbf{x_1}$ of the power rails are to be extended by variables $\mathbf{x_2}$. If on way i a power rail is necessary then $x_{2i} = 0$ otherwise $x_{2i} = 0$. The result is the new constraint (3.23).

While $\mathbf{x_1}$ has technical reasons $\mathbf{x_2}$ is used for economical reasons only. The introduction of $\mathbf{x_2}$ leads to additional constraints:

$$\underbrace{\left(\begin{array}{c|c} \mathbf{A_1} & \mathbf{0} \\ \hline \textbf{-I} & \mathbf{I} \end{array}\right)}_{\mathbf{A}} \cdot \underbrace{\left(\begin{array}{c} \mathbf{x_1} \\ \hline \mathbf{x_2} \end{array}\right)}_{\mathbf{x}} \geq \underbrace{\left(\begin{array}{c} \mathbf{b_1} \\ \hline \mathbf{0} \end{array}\right)}_{\mathbf{b}} \qquad \begin{cases} x_{1i} \in \mathbb{R}_{+} & \wedge \quad 0 \leq x_{1i} \leq 1 \\ x_{2i} \in \mathbb{N}_0 & \wedge \quad 0 \leq x_{2i} \leq 1 \end{cases} \tag{3.23}$$

$$\underbrace{\left(\mathbf{v}^T \cdot diag(\mathbf{len}) \mid \mathbf{w}^T\right)}_{\mathbf{c}} \cdot \left(\begin{array}{c} \mathbf{x_1} \\ \hline \mathbf{x_2} \end{array}\right) \to \min \qquad \begin{cases} v_i \in & \text{invest for 1 meter of} \\ & \text{power rail on way i} \\ w_i \in & \text{basic invest for any} \\ & \text{power rail on way i} \end{cases} \tag{3.24}$$

Because of the introduction of integer variables the economical optimization model is much more complex than the technical model (**??**). If the technical optimization has completed successful the chosen parameters are feasible. This is a precondition to run the economical optimization.

3.4 Charging at bus stops

During the normal operation there is some time at bus stops which can be used for charging. For short this charging policy is called *point charging*. Stop times are often short and equal to the passenger transfer time. But at the terminal points of some bus lines there is often spare time which is much more and can be used for charging as well. However, with a given list of stop times for each bus stop an algorithm can select those, which should be equipped with charging devices.

3.4.1 Point charging model

For each bus $i \in \mathbb{B}$ the stop time ts at each bus stop $k \in \mathbb{S}$ is essential. The stop time matrix $\mathbb{TS}$ keeps all these accumulated values for one global cycle. Because the values strongly depended on the traffic situation we assume some lower bounds.

$$\mathbf{TS} = \begin{pmatrix} ts_{11} & \cdots & ts_{1n} \\ \vdots & \ddots & \vdots \\ ts_{l1} & \cdots & ts_{ln} \end{pmatrix}$$

ts_{ik} : Stop time for bus i
at bus stop k

3.4.2 Technical view on point charging optimization

We now calculate the energy offer at bus stops analog to the [3.19] which is the energy offer for charging on ways. The offer must must be greater equal than the energy demand. Under fulfilling these constraints the number of bus stops which are to be upgraded to charging stations should be minimized.

$$\overbrace{\underbrace{diag(\mathbf{p_{max}}) \cdot \mathbf{TS}}_{A} \cdot \mathbf{x}}^{\text{offer}} \geq \overbrace{\underbrace{\mathbf{SW} \cdot \mathbf{e}^{-} + \mathbf{e_{target}} - \mathbf{e_0}}_{\mathbf{b}}}^{\text{demand}} \qquad \begin{cases} x_i \in \mathbb{N}_0 \\ 0 \leq x_i \leq 1 \end{cases} \tag{3.25}$$

$$(1\,1\,\cdots\,1) \cdot \mathbf{x} \rightarrow \min \tag{3.26}$$

3.4.3 Economical view on point charging optimization

From the economical view there are no additional aspects beyond the technical view.

3.5 Swapping batteries at bus stops

Rather than using a big battery an array of smaller batteries can be used. The advantages are that each battery can be treated at its best operation point and only batteries which run near the lower bound e_{min} are to be switched. We always assume there is enough time for swapping a certain number of batteries at a swapping station. Candidates for swapping stations are bus stops. If a given transportation network does not offer enough feasible locations for battery swapping some extra stops only for battery swapping must be provided. This influences the timetable and the bus schedules and it out of scope of this work.

3.5.1 Swapping battery model

In this paper we do not consider the effect of partially charged batteries and we assume only one type of battery with a capacity of e_b. The objective is to find the bus stops which must be changed over to swapping stations where the number of swapping stations is minimal. For each bus stop i which is a candidate for a swapping station a capacity limit $c_{si} > 0$ is given, otherwise $c_{si} = 0$. The capacity is the number of fully charged batteries which can be delivered during a global cycle time T. The capacity vector $\mathbf{c_s}$ over all bus stops is an input parameter for the optimization problem. Since it is only possible to swap as many batteries as are available the constraint (3.27) must hold.

$$\forall k \in \mathbb{S} : \quad \sum_{i \in \mathbb{B}} y_{ik} \leq c_{sk} \tag{3.27}$$

y_{ik} : Number of batteries to be swapped between bus i and station k in one global cycle

In terms of linear optimization we express this constraint as

$$\begin{pmatrix} -\mathbf{I} & | & \cdots & | & -\mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{y_1} \\ \hline \vdots \\ \hline \mathbf{y}_{|\mathbb{B}|} \end{pmatrix} \geq \mathbf{c_s} \qquad (y_i)_k \in \mathbb{N}_0 \tag{3.28}$$

3.5.2 Technical view on swapping battery optimization

In Order to minimize the number of swapping stations we extend the solution vector by a Boolean vector $\mathbf{z}$. $z_k = 0$ disables the station k and $z_k = 1$ enables the station k. In the first case station k does not offer any batteries, otherwise the assigned capacity c_{sk} is available.

$$\begin{pmatrix} -\mathbf{I} & | & \cdots & | & -\mathbf{I} & | & diag(\mathbf{c_s}) \end{pmatrix} \begin{pmatrix} \mathbf{y_1} \\ \hline \vdots \\ \hline \mathbf{y}_{|\mathbb{B}|} \\ \hline \mathbf{z} \end{pmatrix} \geq \mathbf{0} \qquad \begin{cases} (y_i)_k & \in \mathbb{N}_0 \\ z_i & \in \{0, 1\} \end{cases} \tag{3.29}$$

For each bus i there is a vector $\mathbf{x_i}$ containing over all bus stops the number of batteries to switch. The vector $\mathbf{y}$ over all bus stops indicates if the corresponding stop should be equipped with charging facilities. This leads to the objective function $\sum_i y_i \rightarrow \min$ which minimizes the number of swapping stations. The x vectors are needed to satisfy additional constraints only but do not contribute to the value to minimize.

$$\underbrace{\left(\begin{array}{cccc|c|cccc|cccc} \text{-1} & 0 & \cdots & 0 & \cdots & \text{-1} & 0 & \cdots & 0 & c_1 & 0 & \cdots & 0 \\ 0 & \text{-1} & \cdots & 0 & \cdots & 0 & \text{-1} & \cdots & 0 & 0 & c_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \text{-1} & \cdots & 0 & 0 & \cdots & \text{-1} & 0 & 0 & \cdots & c_{|\mathbb{S}|} \end{array}\right)}_{\mathbf{A_1}} \cdot \underbrace{\left(\begin{array}{c} \mathbf{y_1} \\ \hline \vdots \\ \hline \mathbf{y}_{|\mathbb{B}|} \\ \hline \mathbf{z} \end{array}\right)}_{\mathbf{x}} \geq \underbrace{\left(\begin{array}{c} 0 \\ 0 \\ \vdots \\ 0 \end{array}\right)}_{\mathbf{b_1}} \tag{3.30}$$

The energy e_i for each bus i has to be kept in the range $e_{maxi} \geq e_i \geq e_{mini}$. In contrary to (**??**) which estimates an energy offer only, now two constraints are needed because energy offer and energy demand are aligned immediately by swapping.

$$\mathbf{e_{max}} \geq \mathbf{e_0} - \mathbf{SW} \cdot \mathbf{e}^- + \mathbf{e}^+ \geq \mathbf{e_{min}} \tag{3.31}$$

The buses can take batteries only at swapping station which they pass. It does not matter how often they pass a station but only if they pass. We derive the required Boolean values $\bar{s}_{ik}$ from the schedule matrix SS directly. A multiplication by the battery energy e_b leads to

$$\bar{s}_{ik} = \begin{cases} 0 & : ss_{ik} = 0 \\ e_b & : ss_{ik} > 0 \end{cases} \tag{3.32}$$

$$\mathbf{\bar{S}} = \left(\begin{array}{cccc|c|cccc|cccc} \bar{s}_{11} & \bar{s}_{12} & \cdots & \bar{s}_{1n} & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & \bar{s}_{l1} & \bar{s}_{l2} & \cdots & \bar{s}_{ln} & 0 & 0 & \cdots & 0 \end{array}\right) \tag{3.33}$$

Finally we get the whole optimization problem for the battery swapping problem

$$\underbrace{\left(\begin{array}{c} \mathbf{A_1} \\ \hline \mathbf{\bar{S}} \\ \hline -\mathbf{\bar{S}} \end{array}\right)}_{\mathbf{A}} \cdot \underbrace{\left(\begin{array}{c} \mathbf{y_1} \\ \hline \vdots \\ \hline \mathbf{y}_{|\mathbb{B}|} \\ \hline \mathbf{z} \end{array}\right)}_{\mathbf{x}} \geq \underbrace{\left(\begin{array}{c} \mathbf{0} \\ \hline \mathbf{SW} \cdot \mathbf{e}^- + \mathbf{e_{min}} - \mathbf{e_0} \\ \hline -\mathbf{SW} \cdot \mathbf{e}^- - \mathbf{e_{max}} - \mathbf{e_0} \end{array}\right)}_{\mathbf{b}} \tag{3.34}$$

Basically the objective function $\mathbf{c}$ masks the z-part of the solution vector.

$$\begin{pmatrix} 0 & 0 & \cdots & 0 & | & \cdots & | & 0 & 0 & \cdots & 0 & | & 1 & 1 & \cdots & 1 \end{pmatrix} \begin{pmatrix} \mathbf{y_1} \\ \hline \vdots \\ \hline \mathbf{y_{|\mathbb{B}|}} \\ \hline \mathbf{z} \end{pmatrix} \rightarrow \min \tag{3.35}$$

Rearranging the solution vector to a swapping matrix $\mathbf{SWAP} = \mathbb{N}^{|\mathbb{B}| \times |\mathbb{S}|}$ gives a row of the number of batteries to swap for each bus. The column sum of the swap matrix gives the number of batteries to swap for each swapping station. Elements of zero indicate, that the corresponding bus stops do not need to perform swapping capability.

This result is based on the simple linear model and it gives good start values for a simulation run which should respect some more details for a more realistic mapping to the real world.

3.5.3 Economical view on swapping battery optimization

The result of the technical optimization is a set of all bus stops which must be equipped with swapping facilities and all bus stop are treated as equal candidates. The only difference between candidates might be different capacities. From the economical view there are costs v_i for swapping one battery at station i. Additionally invests w_i for building swapping station i must be taken into account.

$$\begin{pmatrix} v_1 & v_2 & \cdots & v_n & | & \cdots & | & v_1 & v_2 & \cdots & v_n & | & w_1 & w_2 & \cdots & w_n \end{pmatrix} \begin{pmatrix} \mathbf{y_1} \\ \hline \vdots \\ \hline \mathbf{y_{|\mathbb{B}|}} \\ \hline \mathbf{z} \end{pmatrix} \rightarrow \min \tag{3.36}$$

To exclude a bus stop i from offering a swapping functionality the corresponding invest can be set to very high value of invest.

Chapter 4

Economic Methods

4.1 Model of cash flow

4.1.1 Introduction - evaluation of public projects

Implementation of projects in the public sector institutions, like project "CACTUS. Models and Methods for the Evaluation and the Optimal Application of Battery Charging and Switching Technologies for Electric Busses", tends in the direction of the evaluation of socio-economic character. This project, through its allocation, seeks to widely understood improving the living conditions of urban residents and therefore its socio-economic evaluation is one of the key steps in the process of evaluation and selection of optimal solutions [6, p. 27].

CACTUS Project evaluation should be ex-ante evaluation of the three main reasons [16, p. 15] (as cited in [1, p. 58]):

- choice of more effective solutions,
- create an information base for decision-making containing key indicators of evaluation that should be taken into account when choosing a variant solutions
- ensuring communication between participants of the decision making process, including the beneficiaries of the chosen solution.

Ex-ante evaluation is primarily financial. Therefore indicates the potential profits or savings arising from the implementation of a specific variant of the bus service, the type of rolling stock used with an electric motor and a method of exchange and battery charging. According to D. Look financial analysis should use the following parameters [5, pp. 40-46]:

- characteristics of the project results included in clear quantifiable values,

- total expenditures related to the implementation of the project and the cost of its individual products,
- expected date available all effects of the project,
- forecast operating and maintenance costs relating to the project,
- a value, liquidation project
- length of the economic viability of the project products,
- Forecast cost of project financing (loan interest rates, rate of inflation)
- the size of the fiscal (taxes, subsidies),
- schedule allows ordering of costs and benefits in the form of cash flow
- schedule of net cash flows.

To presented general assumptions of the evaluation of public sector projects, refer in Poland, the guidelines contained in the Blue Books, manuals on various areas of transport activity (in this case, public transport). There was presented a method to carry out a cost-benefit (CBA) for the planned investment projects applying for funding from the EU. In the situation described procedures are obligatory. It is recommended to use the basic principles of the manuals for all projects to be financed with public funds. [13, p. 29] [12]

According to the Blue Book for investments in public transport, socio-economic analysis aims to demonstrate that the planned investment variant is justified from a social point of view. Its scope is as follows: [13, pp. 29, 35-41]

- recognition of cash costs for the so-called non-investment option and similarly for the various investment considered variants,
- calculation of net economic benefits of the project expressed the difference of the cost of an investment option and economic costs non-investment variant (Tables 4.1, 4.2 shows the possible types of financial expenses and the costs and economic benefits in projects in the field of public transport),
- adoption of the residual value of the investment project (in Table 4.3 shows the recommended size of the parameter)
- calculation of performance indicators of socio-economic ENPV, ERR, BCR,
- choice of the final variant of the investment project.

Table 4.1: The main categories of financial costs and revenues for investment in public transport [13, p. 29]

Possible financial costs
capital expenditures
operating and maintenance costs (infrastructure)
operating costs (operator)
revenues: from ticket sales other income beyond operating

Table 4.2: The main categories of costs and economic benefits for investment in public transport [13, p. 29]

Possible types of economic costs and benefits
Time costs of existing public transport users
Time costs of car users acquired by public transport
The operating costs of vehicles for current users of private transport who become public transport users
The operating costs of public transport vehicles
The costs of environmental effects
Costs consequences of accidents

Table 4.3: Recommended residual value for certain items of property investment project in public transport (own study based on [13, p. 38])

Name of component of the project	**Residual value [%]**
Bus fleet	0
Tram fleet	20
Subway fleet	25
Rail fleet	30
Transport infrastructure	50

Table 4.4: Steps of adjustment for the fiscal effects [13, p. 35])

Steps	**Steps Adjustment for the fiscal effects**
Step 1	VAT elimination
Step 2	Adjustment for fiscal transfers Capital expenditures (coefficient 0.84 - infrastructure, coefficient of 0.82 - buses, coefficient of 0.86 - trams, trolleybuses) Expenditure on operation (coefficient 0.72 - average)

The analysis should also take into account an adjustment for the fiscal effects. Its steps are presented in Table 4.4.

In view the above reflections , the choice of the method of evaluation of the project CACTUS using tools to analyze cost and benefits are fully justified.

It should be emphasized that this method extends the information field in the selection of a particular variant of the exchange and charge battery of electric buses operating systems of public transport bus network. The first element of the economic analysis was in fact comparable cost method, development in WP2 phase of the project CACTUS.

4.1.2 Total cost of ownership (KCW)

The formula (4.1) described economic present value of variant the project. It is total cost of ownership in the use of electric buses and adapt to their needs and way to exchange and charging the battery less for present value of the proceeds of liquidation:

$$KCW = KA + KO + KI + KZ - PL \; [PLN, EUR] \tag{4.1}$$

KCW	total cost of ownership [PLN, EUR]
KA	present value of the acquisition costs of bus [PLN, EUR]
KO	present value of the operating costs [PLN, EUR]
KI	present value of the infrastructure costs [PLN, EUR]
KZ	present value of the external costs [PLN, EUR]
PL	present value of the proceeds of liquidation [PLN, EUR]

Figure 4.1 shows the general algorithm for calculating the value of (KCW). In further considerations discussed in detail the method of calculation of each of the components of the total cost (KCW).

4.1.3 Present value of the acquisition costs of bus (KA)

For the present value of the cost of electric buses (KA) used to handle traffic on the selected lines of public transport network consisting of cost items are presented in 4.2.

Costs (KA) includes follows components:

- nominal acquisition costs KA_{nom}
- acquisition costs taking into account subsidies KA_{dot}
- self-financed acquisition costs KA_{kw}
- external acquisition costs KA_{kred}

Depending on defining the given elements of cost (KA) are as follows.

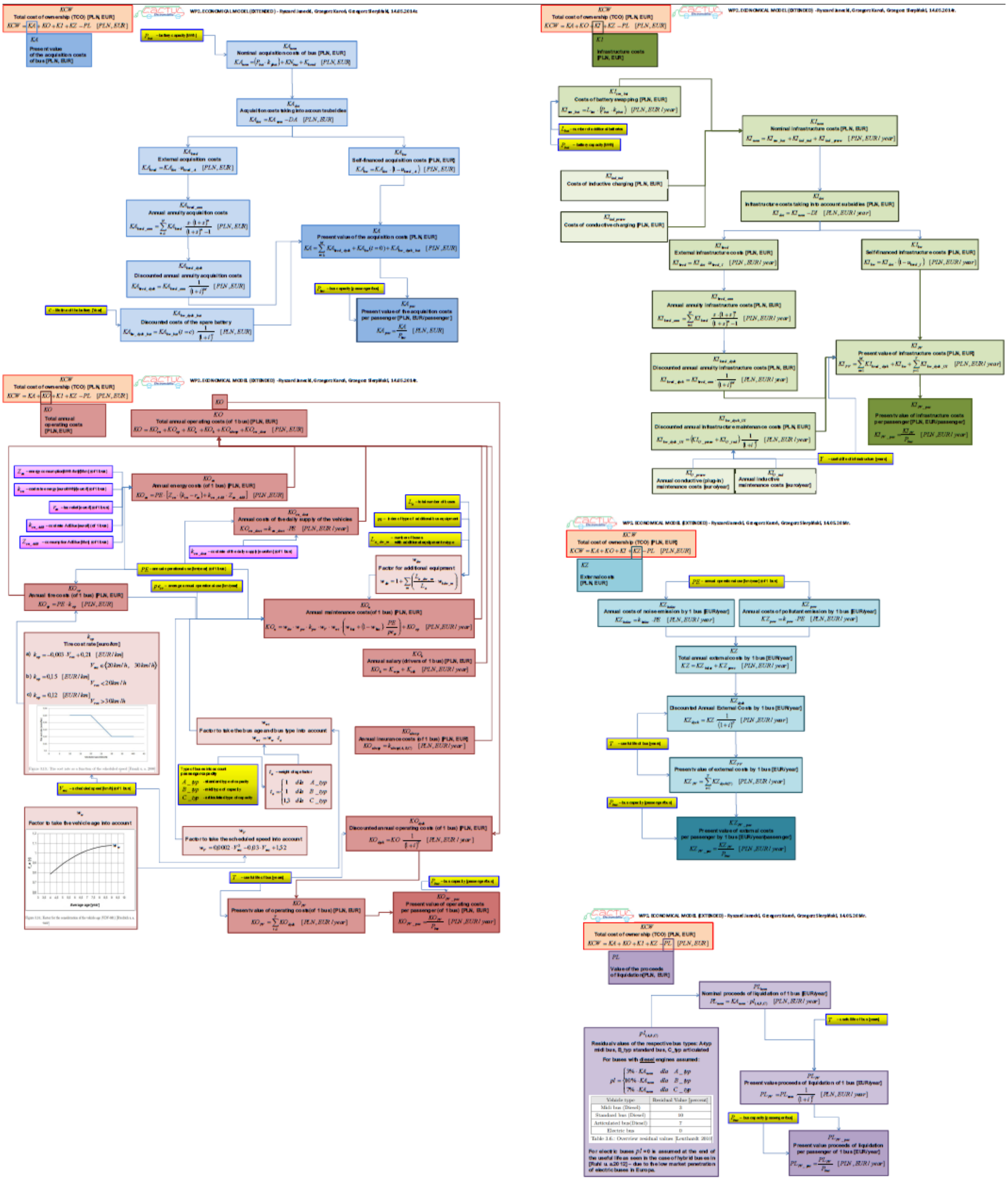


Figure 4.1: Scheme of KCW – total cost of ownership (TCO)

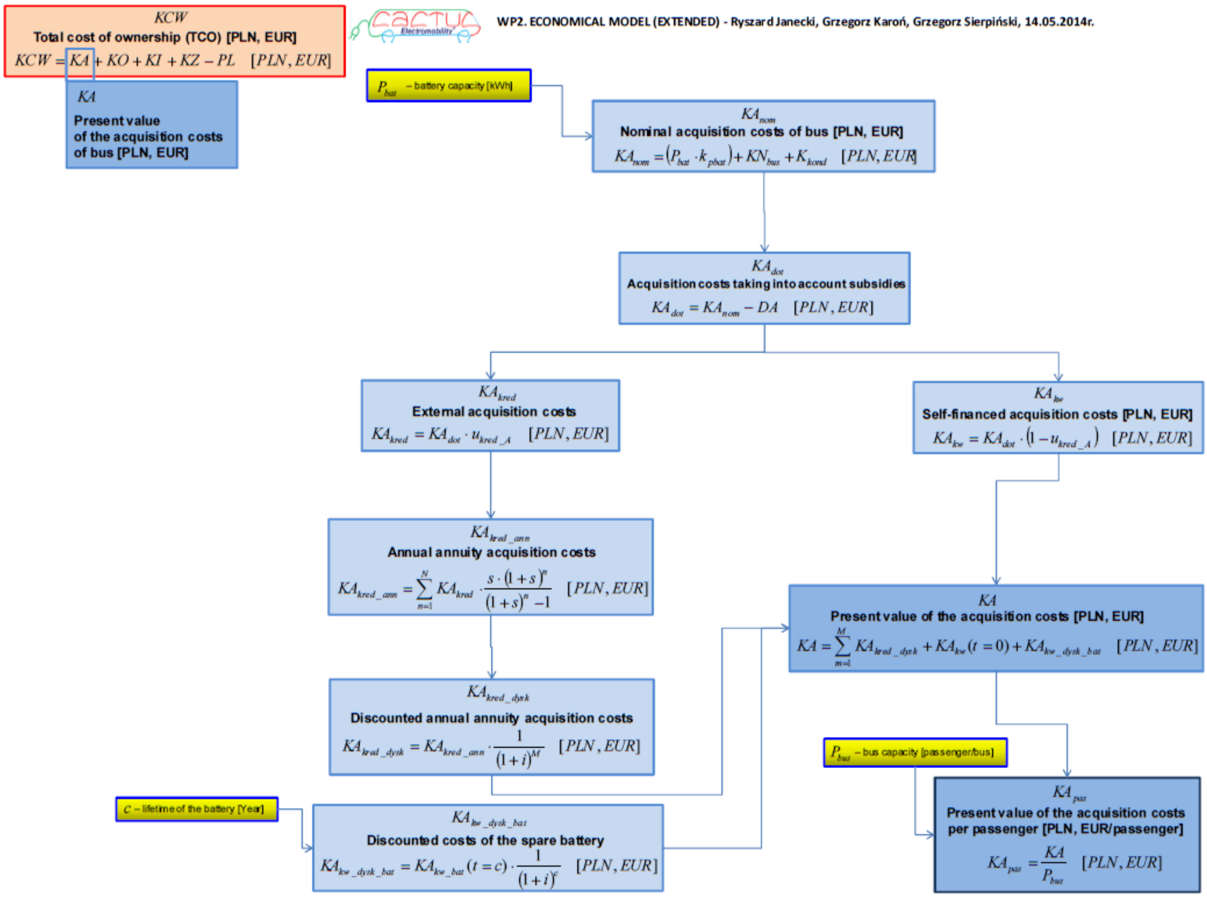


Figure 4.2: Scheme of KA – present value of the acquisition costs

Nominal acquisition costs

$$KA_{nom} = (P_{bat} \cdot k_{pbat}) + KN_{bus} + K_{kond} \; [PLN, EUR] \tag{4.2}$$

KA_{nom}	nominal acquisition costs of bus [PLN, EUR]
P_{bat}	battery capacity [kWh]
k_{pbat}	battery costs [PLN, EUR/kWh]
KN_{bus}	bus costs [PLN, EUR]
K_{kond}	costs for double-layer capacitors [PLN, EUR]

Acquisition costs taking into account subsidies

$$KA_{dot} = KA_{nom} - DA \; [PLN, EUR] \tag{4.3}$$

KA_{dot}	acquisition costs taking into account subsidies [PLN, EUR]
KA_{nom}	nominal acquisition costs [PLN, EUR]
DA	subsidies for bus [PLN, EUR]

Self-financed acquisition costs

$$KA_{kw} = KA_{dot} \cdot (1 - u_{kred\_A}) \;\; [PLN, EUR] \tag{4.4}$$

KA_{kw}	self-financed acquisition costs [PLN, EUR]
KA_{dot}	acquisition costs taking into account subsidies [PLN, EUR]
u_{kred_A}	external finance rate acquisition [-], $u_{kred\_A} \in (0, 1)$

External acquisition costs

$$KA_{kred} = KA_{dot} \cdot u_{kred\_A} \; [PLN, EUR] \tag{4.5}$$

KA_{kred}	external acquisition costs [PLN, EUR]
KA_{dot}	acquisition costs taking into account subsidies [PLN, EUR]
u_{kred_A}	external finance rate acquisition [-],$u_{kred\_A} \in (0, 1)$

Taking into account fiscal conditions of the annual costs of possible variants of of financing the purchase of electric buses and extra batteries represent the relationship (4.6) - (4.8).

Annual annuity acquisition costs

$$KA_{kred\_ann} = \sum_{n=1}^{N} KA_{kred} \cdot \frac{s \cdot (1+s)^n}{(1+s)^n - 1} \; [PLN, EUR] \tag{4.6}$$

KA_{kred_ann}	annual annuity acquisition [PLN, EUR]
KA_{kred}	external acquisition costs [PLN, EUR]
N	credit period [-]
s	loan interest rate [-]
n	number of interest rate [-]

Discounted annual annuity acquisition costs

$$KA_{kred\_dysk} = KA_{kred\_ann} \cdot \frac{1}{(1+i)^M} \; [PLN, EUR] \tag{4.7}$$

KA_{kred_dysk}	discounted annual annuity acquisition [PLN, EUR/Year]
KA_{kred_ann}	annual annuity acquisition [PLN, EUR]
i	market interest rate [-]
M	repayment term acquisition [Year]

Discounted costs of the spare battery

$$KA_{kw\_dysk\_bat} = KA_{kw\_bat}(t=c) \cdot \frac{1}{(1+i)^c} \; [PLN, EUR] \tag{4.8}$$

$KA_{kw\_dysk\_bat}$	discounted costs of the spare battery [PLN, EUR/Year]
$KA_{kw\_bat}(t=c)$	self-financed acquisition costs of the spare battery [PLN, EUR]
i	market interest rate [-]
c	lifetime of the battery [Year]

Present value of the acquisition costs

$$KA = \sum_{m=1}^{M} KA_{kred\_dysk} + KA_{kw}(t=0) + KA_{kw\_dysk\_bat} \; [PLN, EUR] \tag{4.9}$$

KA	present value of the acquisition costs [PLN, EUR]
KA_{kred_dysk}	discounted annual annuity acquisition [PLN, EUR/Year]
$KA_{kw}(t=0)$	self-financed acquisition costs at the beginning of the investment [PLN, EUR]
$KA_{kw\_dysk\_bat}$	discounted costs of the spare battery [PLN, EUR/Year]
M	repayment term acquisition [Year]

Taking as a point of reference costs for the one passenger on the bus, the resulting value of this ratio is expressed as relationship:

Present value of the acquisition costs per passenger

$$KA_{pas} = \frac{KA}{P_{bus}} \; [PLN, EUR] \qquad (4.10)$$

KA_{pas}	present value of the acquisition costs per passenger [PLN, EUR]/passenger
KA	present value of the acquisition costs [PLN, EUR]
P_{bus}	bus capacity [passenger/bus]

4.1.4 Total annual operating costs (KO)

The value of the operating costs should be estimated for both non-investment variant (existing state) and for all variants of solutions using electric buses. It is particularly difficult to estimate the projected operating and maintenance costs over time.

Operating expenses (KO) taken into account in the evaluation of individual variants, consists of seven components. This represents a relationship (4.11) and Figure 4.3.

$$KO = KO_{en} + KO_{op} + KO_{u} + KO_{k} + KO_{ubezp} + KO_{en\_dost} \; [PLN, EUR] \qquad (4.11)$$

KO	total annual operating costs,
KO_{en}	annual energy costs,
KO_{op}	annual tire costs,
KO_{u}	annual maintenance costs,
KO_{k}	annual salary,
KO_{ubezp}	annual insurance costs,
KO_{en_dost}	annual costs for the daily supply of the buses

The individual components of operating cost (KO) is calculated from (4.12) - (4.22). The reference point is the one-year period and one operated bus.

Therefore, expenses related to energy and fuel are as follows:

Annual energy costs

$$KO_{en} = PE \cdot \lfloor Z_{en} \cdot (k_{en} - r_{en}) + k_{en\_AdB} \cdot Z_{en\_AdB} \rfloor \; [PLN, EUR] \tag{4.12}$$

KO_{en}	annual energy costs,
PE	annual operational use [km/year],
Z_{en}	energy consumption[kWh/km]/[l/km],
k_{en}	cost rate energy [euro/kWh]/[euro/l],
r_{en}	tax relief [euro/l],
k_{en_AdB}	cost rate AdBlue [euro/l],
Z_{en_AdB}	consumption AdBlue [l/km],

Cost of tires can be estimated from the relationship:

Annual tire costs

$$KO_{op} = PE \cdot k_{op} \; [PLN, EUR] \tag{4.13}$$

$$k_{op} = -0.003 \cdot V_{roz} + 0.21; V_{roz} \in \langle 20km/h, 30km/h \rangle \; [PLN, EUR] \tag{4.14}$$

KO_{op}	annual tire costs,
PE	annual operational use [km/year],
k_{op}	tire cost rate [euro/km],
V_{roz}	scheduled speed [km/h],

Similarly, the calculated costs of fuel supply:

Annual costs of the daily supply of the vehicles

$$KO_{en\_dost} = k_{en\_dost} \cdot PE \; [PLN, EUR/year] \tag{4.15}$$

KO_{en_dost}	annual costs of the daily supply of the vehicles
k_{en_dost}	cost rate of the daily supply [euro/km]
PE	annual operational use [km/year]

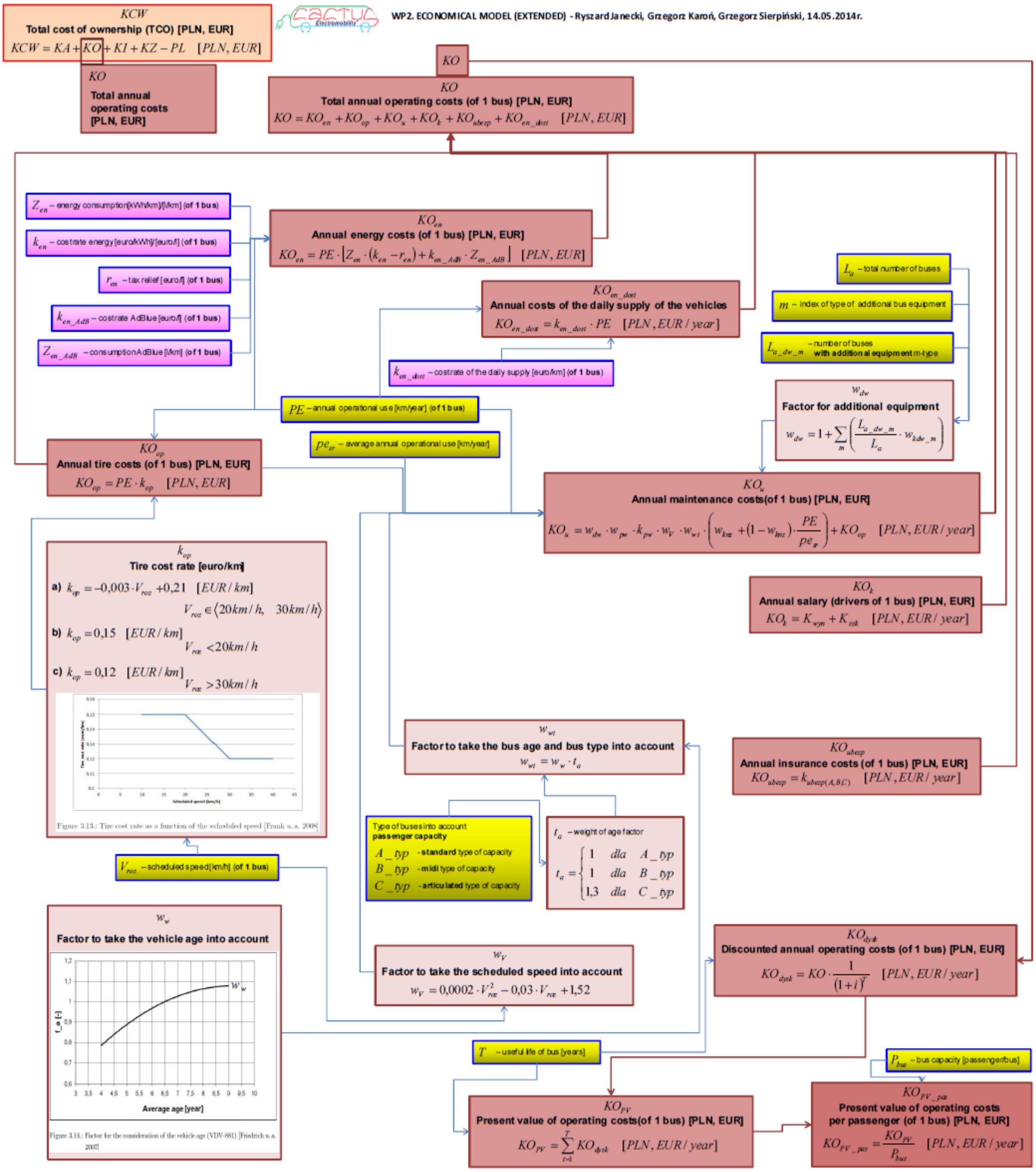


Figure 4.3: Scheme of KO – present value of the operating costs

His second component of operating costs are the annual costs of maintaining a single bus. Typically, they include all costs to ensure the safety of vehicles in technical terms and their's availability to daily operations. These costs express the relations (4.16) - (4.20).

Annual maintenance costs

$$KO_u = w_{dw} \cdot w_{pw} \cdot k_{pw} \cdot w_V \cdot w_{wt} \cdot \left(w_{knz} + (1 - w_{knz} \cdot \frac{PE}{pe_{sr}} \right) \\ +KO_{op} \; [PLN, EUR/year] \tag{4.16}$$

$$w_{dw} = 1 + \sum_m \left(\frac{L_{a\_dw\_m}}{L_a} \cdot w_{kdw\_m} \right) \tag{4.17}$$

$$w_v = 0.0002 \cdot_{roz}^{2} - 0.03 \cdot V_{roz} + 1.52 \tag{4.18}$$

$$w_{wt} = w_w \cdot t_a \tag{4.19}$$

$$t_a = \begin{cases} 1 & dla\ A\_typ \\ 1 & dla\ B\_typ \\ 1.3 & dla\ C\_typ \end{cases} \tag{4.20}$$

KO_u	annual maintenance costs,
w_{dw}	factor for additional equipment,
$L_{a\_dw\_m}$	number of buses with additional equipment m-type,
m	type of additional bus equipment,
L_a	total number of buses,
w_{kdw_m}	extra costs of additional bus equipment m-type per bus,
w_{pw}	staff index [man-year/vehicle/year],
k_{pw}	cost rate workshop staff [euro/man-year],
w_V	factor to take the scheduled speed into account,
V_{roz}	scheduled speed [km/h],
w_{wt}	factor to take the bus age and bus type into account,
A_typ, B_typ, C_typ	type of buses into account passenger capacity: standard type of capacity, midi type of capacity, articulated type of capacity,
w_w	factor to take the vehicle age into account (see Figure 4.4)
t_a	weight of age factor,
w_{knz}	share of maintenance costs independent of the operational use; $w_{knz} \in (0, 1)$,
$(1 - w_{knz})$	share of maintenance costs dependent of the operational use;
PE	annual operational use [km/year],
pe_{sr}	average annual operational use [km/year],
KO_{op}	annual tire costs,

Operating costs also consist of the costs associated with human resources and the cost of vehicle insurance. Can be estimated using (4.21) and (4.22).

Annual salary

$$KO_k = K_{wyn} + K_{szk}\ [PLN, EUR/year] \tag{4.21}$$

KO_k	annual salary [EUR/year],
K_{wyn}	cost rate for the bus driver [EUR/year],
K_{szk}	costs for further education concerning operation, maintenance and repair of electric buses [EUR/year],

Annual insurance costs

$$KO_{ubezp} = k_{ubezp(A,B,C)}\ [PLN, EUR/year] \tag{4.22}$$

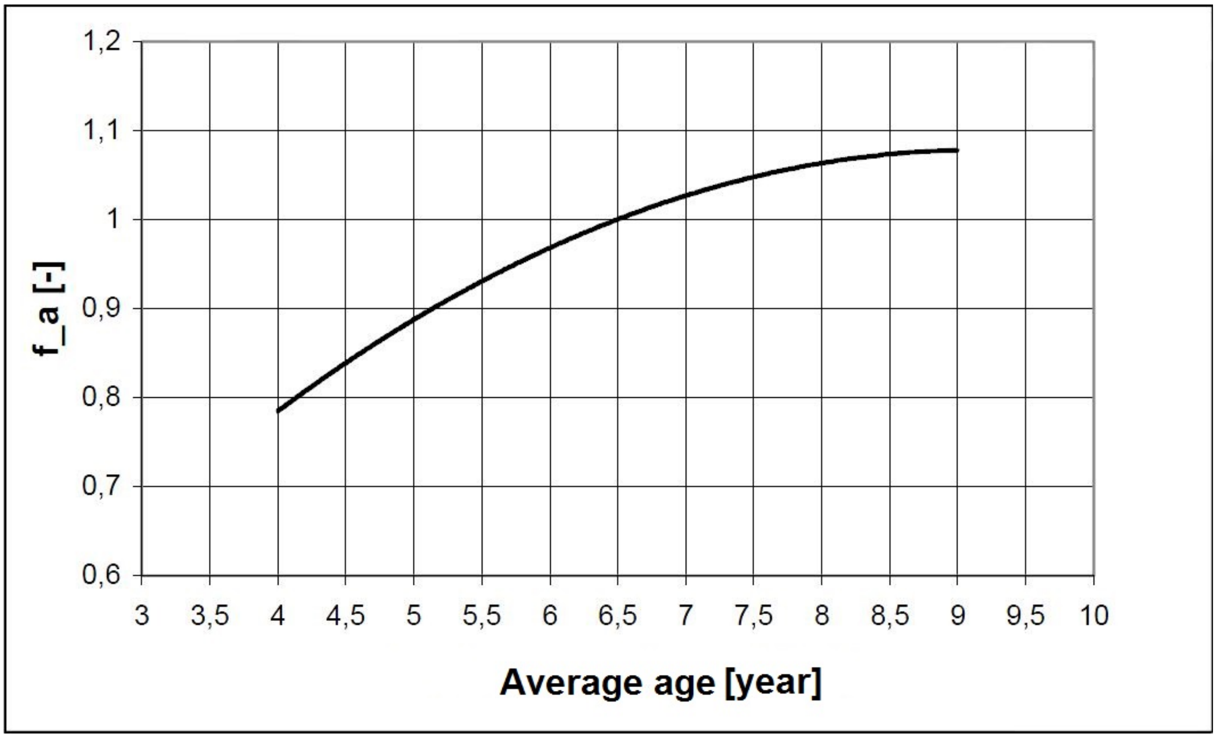


Figure 4.4: Factor for the consideration fo the vehicle age (VDV-881) [11] cited in [2]

KO_{ubezp}	annual insurance cost of one bus,
$k_{ubezp(A,B,C)}$	insurance cost rate with the respective bus type: A_typ, B_typ, C_typ standard type of capacity, midi type of capacity, articulated type of capacity,

Summing up the various sub-categories of costs determined by total operating expenses described in equation (4.11).

Further analysis of operating costs requires consideration of the fiscal factor, and thus estimate the discounted annual operating costs (4.11).

Discounted annual operating costs

$$KO_{dysk} = KO \cdot \frac{1}{(1+i)^T} \; [PLN, EUR/year] \tag{4.23}$$

KO_{dysk}	discounted annual operating costs,
KO	total annual operating costs,
i	market interest rate [-]
T	useful life of bus [years].

Necessary in the method of evaluation of considered variants of solutions: the absolute value of the operating costs in period t, t = 1, 2, ... T, and their unit value attributable to one passenger is calculated as follow:

Present value of operating costs

$$KO_{PV} = \sum_{t=1}^{T} KO_{dysk} \; [PLN, EUR/year] \tag{4.24}$$

KO_{PV}	Present value of operating costs,
KO_{dysk}	discounted annual operating costs,
i	market interest rate [-]
T	useful life of bus [years].

Present value of operating costs per passenger

$$KO_{PV\_pas} = \frac{KO_{PV}}{P_{bus}} \; [PLN, EUR/year] \tag{4.25}$$

KO_{PV_pas}	present value of the operating costs per passenger [euro/passenger],
KO_{PV}	present value of the operating costs,
P_{bus}	bus capacity [passenger/bus]

4.1.5 Infrastructure costs (KI)

Infrastructure costs depends of many parameters. The most important of them were included in ths method. The structure of the infrastructure costs was shown on Figure 4.6: The nominal infrastructure costs are calculated by adding three types of costs (see on equation (4.26):

- costs of battery swapping;
- costs of inductive charging;
- costs of conductive charging.

The first of these values depends on battery capacity and of course number of batteries (additional). Next values includes different charging technology and strategy of charging. In this method possibility of subsidies (4.28) and credits cost (4.31, 4.32) were also included.

At the end of last section present value of infrastructure costs, as well as "per passenger" were estimated (equations (4.34) and (4.35)).

Nominal infrastructure costs

At the start of model of cash flow formula to calculate the nominal infrastructure costs was shown (4.26). It is sum of three components:

$$KI_{nom} = KI_{zm\_bat} + KI_{lad\_ind} + KI_{lad\_przew}\ [PLN, EUR/year] \tag{4.26}$$

KI_{nom}	nominal infrastructure costs [euro],
KI_{zm_bat}	costs of battery swapping to meet the range requirement of up to 300 km per day [euro],
KI_{lad_ind}	costs of inductive charging [euro],
KI_{lad_przew}	costs of conductive charging (plug-in) [euro],

First of this elements was described in equation (4.27). Costs of battery swapping depends among others on battery capacity, but number of additional batteries are also important.

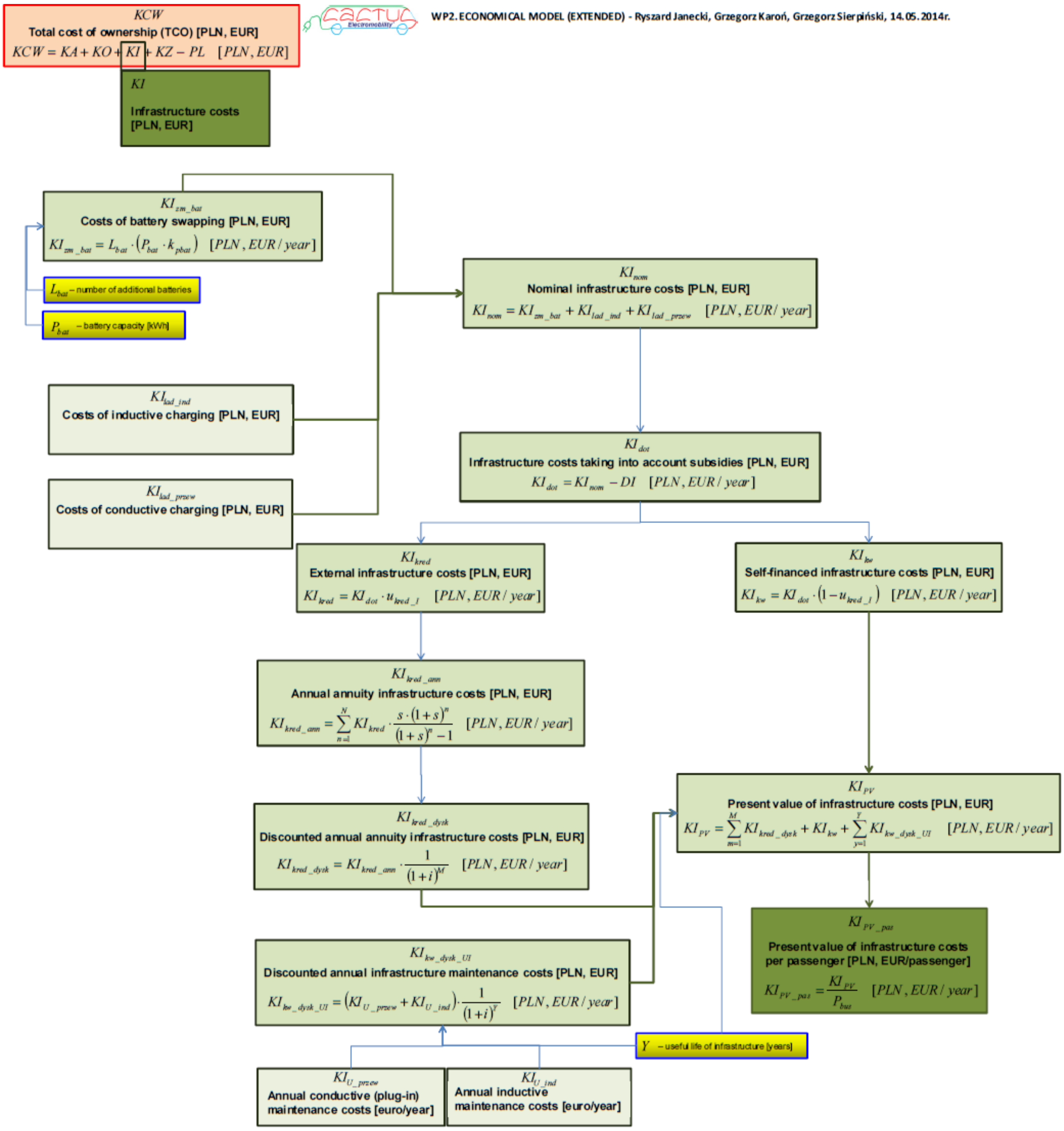


Figure 4.5: Scheme of KI – present value of the infrastructure costs

Costs of battery swapping

$$KI_{zm\_bat} = L_{bat} \cdot (P_{bat} \cdot k_{pbat}) \ [PLN, EUR/year] \tag{4.27}$$

KI_{zm_bat} costs of battery swapping [EUR],
L_{bat} number of additional batteries,
P_{bat} battery capacity [kWh],
k_{pbat} costs per unit of battery capacity [euro/kWh],

In the next few formulas included items such as subsidies, loans, credit period and some annuity costs (equations 4.28 - 4.33). It allows to estimate present value of infrastructure costs (4.34). At the end of this part of method present value of infrastructure costs per passenger was shown (4.35). This value was possible to calculate by including bus capacity (in passengers).

Infrastructure costs taking into account subsidies

$$KI_{dot} = KI_{nom} - DI \ [PLN, EUR/year] \tag{4.28}$$

KI_{dot} nominal infrastructure costs [euro],
KI_{nom} infrastructure costs taking into account subsidies [euro],
DI subsidies for infrastructure [PLN, EUR].

Self-financed infrastructure costs

$$KI_{kw} = KI_{dot} \cdot (1 - u_{kred\_I}) \ [PLN, EUR/year] \tag{4.29}$$

KI_{kw} self-financed infrastructure costs [PLN, EUR],
KI_{dot} infrastructure costs taking into account subsidies [euro],
u_{kred_I} external finance rate infrastructure [-], $u_{kred\_I} \in (0, 1)$

External infrastructure costs

$$KI_{kred} = KI_{dot} \cdot u_{kred\_I} \ [PLN, EUR/year] \tag{4.30}$$

KI_{kred} external infrastructure costs [PLN, EUR],
KI_{dot} infrastructure costs taking into account subsidies [euro],
u_{kred_I} external finance rate infrastructure [-], $u_{kred\_I} \in (0, 1)$

Annual annuity infrastructure costs

$$KI_{kred\_ann} = \sum_{n=1}^{N} KI_{kred} \cdot \frac{s \cdot (1+s)^n}{(1+s)^n - 1} \; [PLN, EUR/year] \tag{4.31}$$

KI_{kred_ann}	annual annuity infrastructure costs [PLN, EUR],
KI_{kred}	external infrastructure costs [PLN, EUR],
s	loan interest rate [-]
N	credit period [-]
n	number of interest rate [-]

Discounted annual annuity infrastructure costs

$$KI_{kred\_dysk} = KI_{kred\_ann} \cdot \frac{1}{(1+i)^M} \; [PLN, EUR/year] \tag{4.32}$$

KI_{kred_dysk}	discounted annual annuity infrastructure costs [PLN, EUR/Year]
KI_{kred_ann}	annual annuity infrastructure costs [PLN, EUR],
i	market interest rate [-]
M	repayment term infrastructure [Year]

Discounted annual infrastructure maintenance costs

$$KI_{KW\_dysk\_UI} = \left(KI_{U\_przew} + KI_{U\_ind} \cdot \frac{1}{(1+i)^Y} \right) \; [PLN, EUR/year] \tag{4.33}$$

$KI_{KW\_dysk\_UI}$	discounted annual infrastructure maintenance costs,
KI_{U_ind}	annual conductive maintenance costs [euro/year] (plug-in),
KI_{U_przew}	annual inductive maintenance costs [euro/year],
i	market interest rate [-]
Y	useful life of infrastructure [year].

Finally, the infrastructure cost per passenger was calculated (equation (4.35). This present value allow to specify one of the cost elements. In the case of actual transport cost will be inversely proportional to the number of passengers (and dependent on demand).

Present value of infrastructure costs

$$KI_{PV} = \sum_{m=1}^{M} KI_{kred\_dysk} + KI_{kw} + \sum_{y=1}^{Y} KI_{kw\_dysk\_UI} \; [PLN, EUR/year] \tag{4.34}$$

KI_{PV}	present value of infrastructure costs,
KI_{kred_dysk}	discounted annual annuity infrastructure costs [PLN, EUR/Year]
KI_{kw}	self-financed infrastructure costs [PLN, EUR],
$KI_{kw\_dysk\_UI}$	discounted annual infrastructure maintenance costs,
M	repayment term infrastructure [Year]
Y	useful life of infrastructure [year].

Present value of infrastructure costs per passenger

$$KI_{PV\_pas} = \frac{KI_{PV}}{P_{bus}} \; [PLN, EUR/year] \tag{4.35}$$

KI_{PV_pas}	present value of infrastructure costs per passenger [euro/passenger],
KI_{PV}	present value of infrastructure costs,
P_{bus}	bus capacity [passenger/bus]

4.1.6 External costs (KZ)

This section allows to estimate external costs in economic model. The key of transport policy in EU is to minimize environmental impact of transport. This means some changes on fields of ransport infrastructure and construction of maens of transport (for example "polluter pays" principle).

The procedure to calculate final two values of external cost – present value by 1 bus and per passenger was shown on Figure 4.6. This component of total costs includes among others noise and pollutant emission costs (equation 4.36 and 4.37). At the start annual operational use is important as input data. Total annual external costs is estimating by sum annual costs of noise emission and annual costs of pollutant emission (4.37).

Annual noise emission costs

$$KZ_{halas} = k_{halas} \cdot PE \; [PLN, EUR/year] \tag{4.36}$$

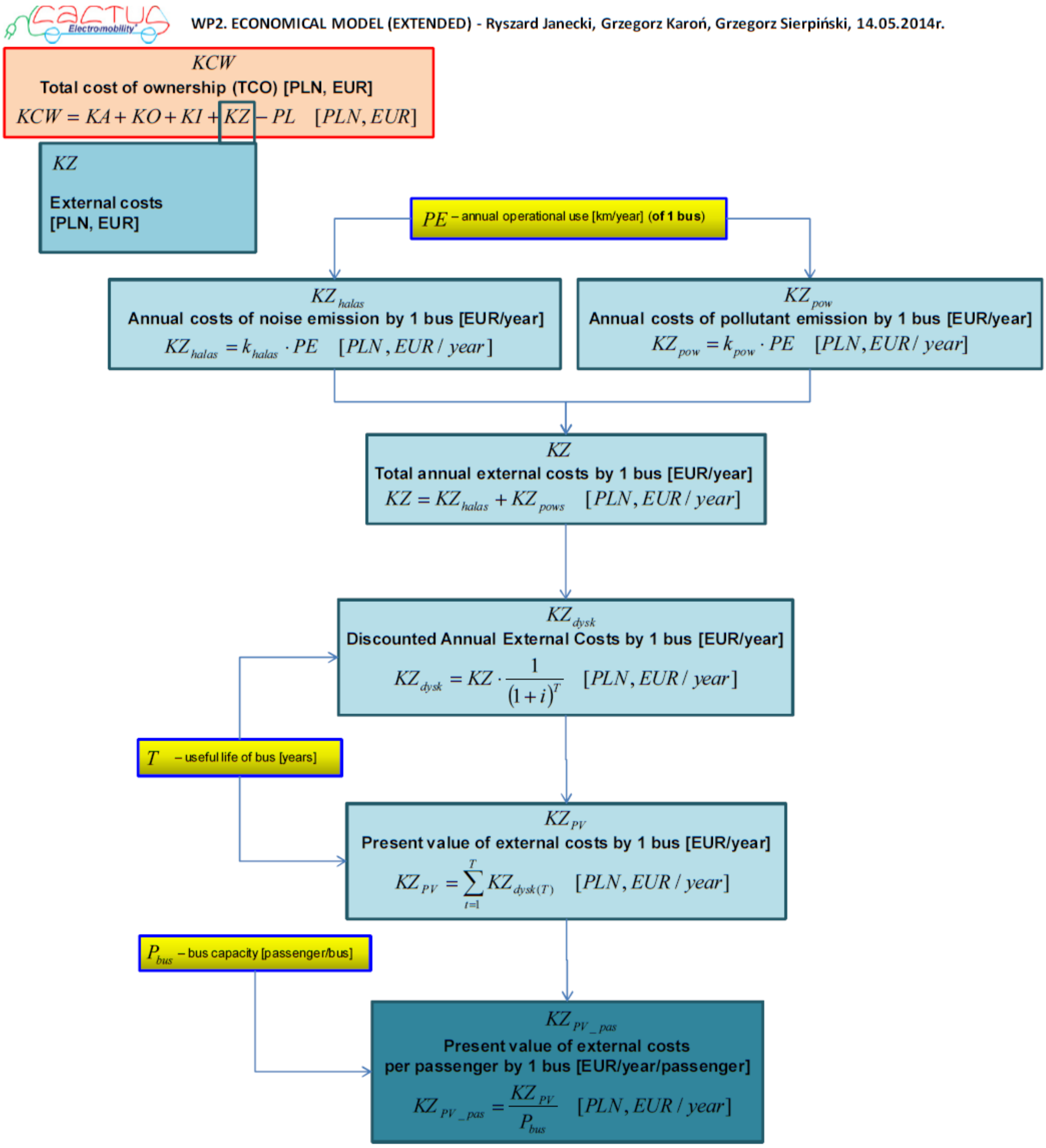


Figure 4.6: Scheme of KZ – present value of the external costs

KZ_{halas} annual costs of noise emission by 1 bus [EUR/year],
k_{halas} cost rate of noise emission by 1 bus [euro/km],
PE annual operational use [km/year],

Annual pollutant emission costs

$$KZ_{pow} = k_{pow} \cdot PE \; [PLN, EUR/year] \tag{4.37}$$

KZ_{pow} annual costs of pollutant emission by 1 bus [EUR/year],
k_{pow} cost rate of pollutant by 1 bus [euro/km],
PE annual operational use [km/year],

Total annual external costs

$$KZ = KZ_{halas} + KZ_{pow} \; [PLN, EUR/year] \tag{4.38}$$

KZ total annual external costs [EUR/year],
KZ_{halas} annual costs of noise emission by 1 bus [EUR/year],
KZ_{pow} annual costs of pollutant emission by 1 bus [EUR/year],

Discounted Annual External Costs

$$KZ_{dysk} = KZ \cdot \frac{1}{(1+i)^T} \; [PLN, EUR/year] \tag{4.39}$$

KZ_{dysk} discounted annual external costs [EUR/year],
KZ total annual external costs [EUR/year],
i market interest rate [-]
T useful life of bus [years].

Present value of external costs

$$KZ_{PV} = \sum_{t=1}^{T} KZ_{dysk(T)} \; [PLN, EUR/year] \tag{4.40}$$

KZ_{PV} present value of the external costs [euro],
KZ_{dysk} discounted annual external costs [EUR/year],
T useful life of bus [years].

Finally, the external costs per passenger was calculated (equation 4.41). This present value allow to specify one of the cost elements. In the case of actual transport cost will be inversely proportional to the number of passengers (and dependent on demand).

Present value of external costs per passenger

$$KZ_{PV\_pas} = \frac{KZ_{PV}}{P_{bus}} \; [PLN, EUR/year] \tag{4.41}$$

KZ_{PV_pas} present value of the external costs per passenger [euro/passenger],
KZ_{PV} present value of the external costs [euro],
P_{bus} bus capacity [passenger/bus]

4.1.7 Present value of the proceeds of liquidation (PL)

This procedure starts with the calculation of nominal proceeds of liquidation (equation 4.42). Three types of diesel buses in this method was specified (standard type of capacity, midi type of capacity, articulated type of capacity) and one electric bus also. It is important to assume residual valuesof the respective bus type. Next step is to estimate present value proceeds of liquidation by using useful life of bus and market interest rate (4.43). Finally, as in previous cases, this value divides by bus capacity to estimate present value proceeds of liquidation per passenger (4.44).

Nominal proceeds of liquidation

$$PL_{nom} = KA_{nom} \cdot pl_{(A,B,C)} \; [PLN, EUR/year] \tag{4.42}$$

PL_{nom} nominal proceeds of liquidation,
KA_{nom} nominal acquisition costs [PLN, EUR]
$pl_{(A,B,C)}$ residual values of the respective bus types:
A_typ midi bus, B_typ standard bus, C_typ articulated,

For buses with diesel engines assumed:

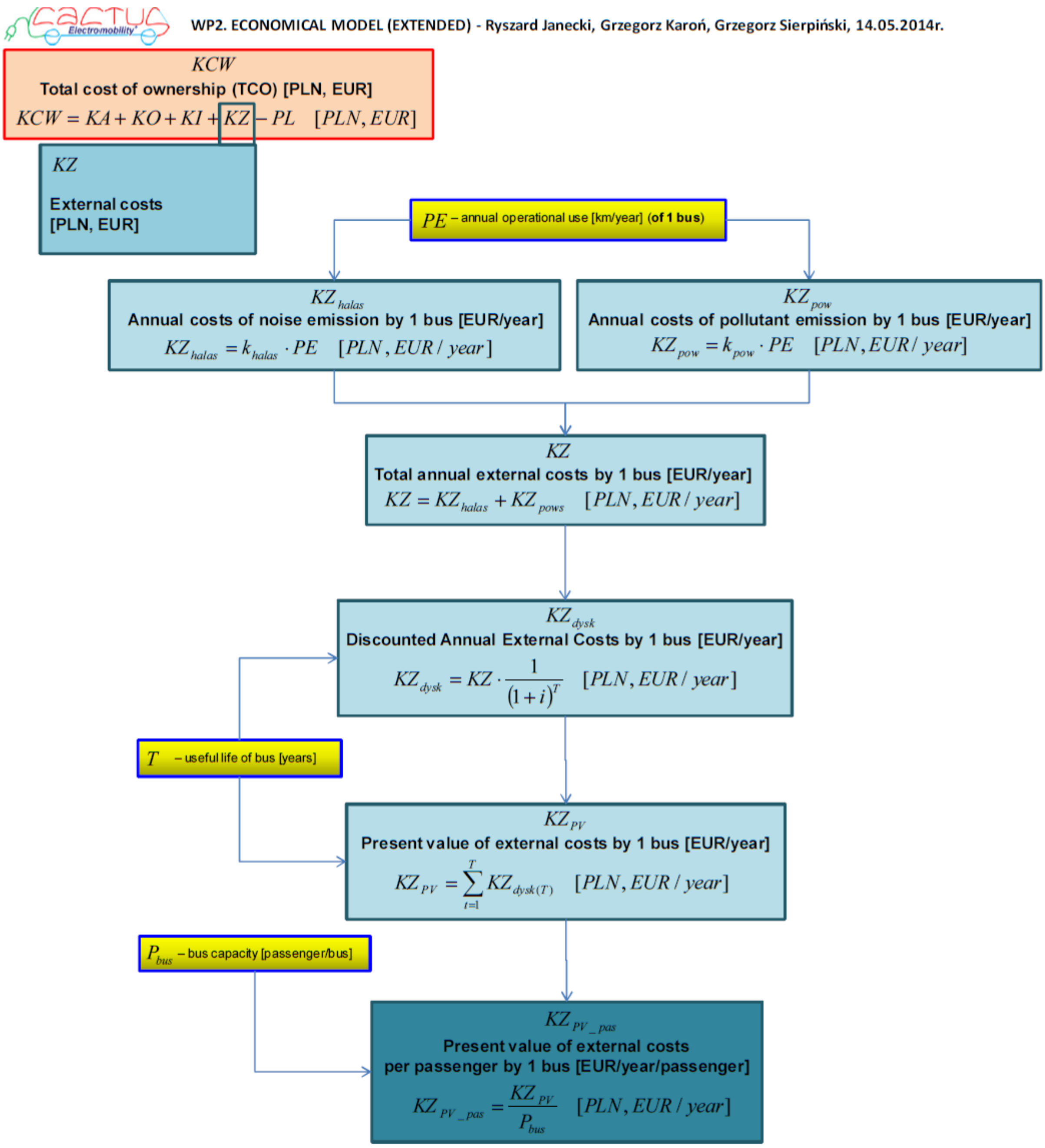


Figure 4.7: Scheme of PL – present value of the proceeds of liquidation

Table 4.5: Overview residual values [14] cited in [2]

Vehicle type	Residual Value [percent]
Midi bus (Diesel)	3
Standard bus (Diesel)	10
Articulated bus (Diesel)	7
Electric bus	0

$$\begin{cases} 3\% \cdot KA_{nom} & dla\ A\_typ \\ 10\% \cdot KA_{nom} & dla\ B\_typ \\ 7\% \cdot KA_{nom} & dla\ C\_typ \end{cases} \tag{4.43}$$

For electric buses $pl_{(A,B,C)} = 0$ is assumed at the end of the useful life as seen in the case of hybrid buses in [17] – due to the low market penetration of electric buses in Europa.

Present value proceeds of liquidation

$$PL_{PV} = PL_{nom} \cdot \frac{1}{(1+i)^T}\ [PLN, EUR/year] \tag{4.44}$$

PL_{PV}	present value proceeds of liquidation,
PL_{nom}	nominal proceeds of liquidation,
i	market interest rate [-]
T	useful life of bus [years].

Finally, the present value proceeds of liquidation per passenger was calculated (equation 4.45). This present value allow to specify one of the cost elements. In the case of actual transport cost will be inversely proportional to the number of passengers (and dependent on demand).

Present value proceeds of liquidation per passenger

$$PL_{PV\_pas} = \frac{PL_{PV}}{P_{bus}}\ [PLN, EUR/year] \tag{4.45}$$

PL_{PV_pas}	present value proceeds of liquidation per passenger,
PL_{PV}	present value proceeds of liquidation,
P_{bus}	bus capacity [passenger/bus]

4.2 Model of cash flow – model implemented in XML

```
<?xml version="1.0" encoding="utf-8"?>
<CACTUS>
  <Format name="CACTUS" subname="Economy_flow_KCW" type="input"
   version="0.1"/>
  <Company name="Przedsiebiorstwo Komunikacji Miejskiej Sp. z o.o. w
   Sosnowcu" short="PKM Sosnowiec"/>
  <KA> // present value of the acquisition costs of bus
    <id="KA" name="present value of the acquisition costs of bus"
     PLN="" EUR="" />
  <KO> // present value of the operating costs
    <id="KO" name="present value of the operating costs" PLN=""
     EUR="" />
  <KI> // present value of the infrastructure costs
    <id="KI" name="present value of the infrastructure costs"
     PLN="" EUR="" />
  <KZ> // present value of the external costs
    <id="KZ" name="present value of the external costs" PLN=""
     EUR="" />
  <PL> // present value of the proceeds of liquidation
    <id="PL" name="present value of the proceeds of liquidation"
     PLN="" EUR="" />
</CACTUS>
```

4.3 Answering the Questions from Deliverable 1.1

This section explains in detail how the questions from Deliverable 1.1 can be answered by using economical methods.

General question in economic part of project is:

How much is the economic drawback or benefit when pure electric busses would be used instead of diesel busses, natural gas busses or hybrid busses?

The economic model includes two-stage evaluation of considered alternative solutions, the aim of which is to demonstrate that a variant of the investment (electric buses on selected lines on network of public transport system available with optimal technical and operating system of exchange and battery charging drive) is justified:

- from an economic point of view - the method of comparative cost (step WP2)

- from the social point of view - a method that enables a cost-benefit calculation of indicators of social efficiency ENPV, ERR and BCR (stage WP3).

The proposed model allows therefore the answer to the question about the effectiveness of a given variant solutions.

4.3.1 Energy Consumption

What will it cost to purchase the required number of battery-powered busses (3.1a)?

In the economic model of the present value of the total cost of the investment variant (KCW), one of the components of that cost is a category of the acquisition costs of electric bus (KA), expressed in current prices reflecting the fiscal options.

What will be the investments in infrastructure and technical facilities for electric busses (3.1b)?

The infrastructure costs depend on type of charging technology. The conditions of electric powered buses operation require providing them an route battery charging. A few variants of charging rechargeable batteries are possible. For example charging with use of pantograph on selected bus stop or plug-in charging (in depots). Other possibility of energy exchange is efficient replacement of the whole battery in replacement points in a bus route (if construction of such battery will be appropriate). The investment costs of the vehicles and the infrastructure (e.g. buildings, service stations, battery charging/exchanging stations) also depends on government funding, interest on loan, runtime of the loan, discounts etc. Therefore in this method possibility of subsidies was included (equation 4.28). At the end of this part of method present value of infrastructure costs and also in option "per passenger" were described (equations 4.34 and 4.35).

4.3.2 Operational Costs

What are the operating costs of the electric busses to procure the electrical energy (3.2a)?

The relation between some elements helps to estimation the operating costs of the electric buses was shown at Figure 4.5. General formula for estimating of total annual operating costs is given by equation (4.11). This calculation method also gives the answer to the question about the value of each operating costs, such as:

annual energy cost (4.12), annual tire cost (4.13), annual maintenance cost (4.15), annual salary (4.16) and insurance cost (4.21). Equation to calculate annual costs of the daily supply of the vehicles there is also shown (4.22). What is important, sum of all of the above elements give exactly total annual operating costs.

What are the maintenance costs for the electric busses and the infrastructure (3.2b)?

Answer this question give equations (4.15) and, as nominal of infrastructure cost, equation (4.26).

Maintrance cost is more complex due to many parameters. Some of them are time-dependent – like factor to take the vehicle age into account. Type of buses is also important. Three types of buses in this method was specified (standard type of capacity, midi type of capacity, articulated type of capacity).

What is the difference to conventional busses if they are equipped with fuel saving systems (e.g. stop & go) (3.2c)?

Comparison of operationg cost between conventional buses and some other types could be realize by using economic model described in WP2. In general that model allows to calculate variable and fixed costs. There are some formulas included parameters like costs of: fuel oils, technical lubricant, electric energy for traction, tires, repairs, renovations of rolling stock and infrastructure, materials for operational work and others. Determination of costs for each of the types of buses allow obtaining cost differences between them.

Chapter 5

Ecological Methods

5.1 Air pollution and greenhouse gases

The energy consumption in kWh from all considered busses over the considered period of time is known from the simulation (see Chapter 2). By this and including efficiency loses (e.g. diesel engine [9]) the demand of different energy carriers is calculated by equation (5.1).

$$M = \frac{E}{w_{Energy} \cdot \eta} \tag{5.1}$$

M	total amount of energy source (e.g. diesel [l], electricity [kWh])
E	total amount of energy needed for moving the busses [kWh]
w_{Energy}	energy density (electricity is assumed as an energy density of 1)
η	efficiency of the engine

The energy sources considered within the CACTUS project are:

- Diesel (Energy density: 9.7 kWh/l [8])
- Natural gas (Energy density: 7 kWh/l [15])
- Electricity, German Mix 2013 (25.8% brown coal, 23.4% Renewables, 19.7% black coal, 15.4% nuclear, 10.5% gas, 5.2% other [7])
- Electricity, 100% Renewables

While the energy is consumed, local and global air pollutants and greenhouse gases are produced depending on the type of engine. When a pure electrical engine is used no air pollutants and greenhouse gases are emitted locally but globally. When a diesel engine

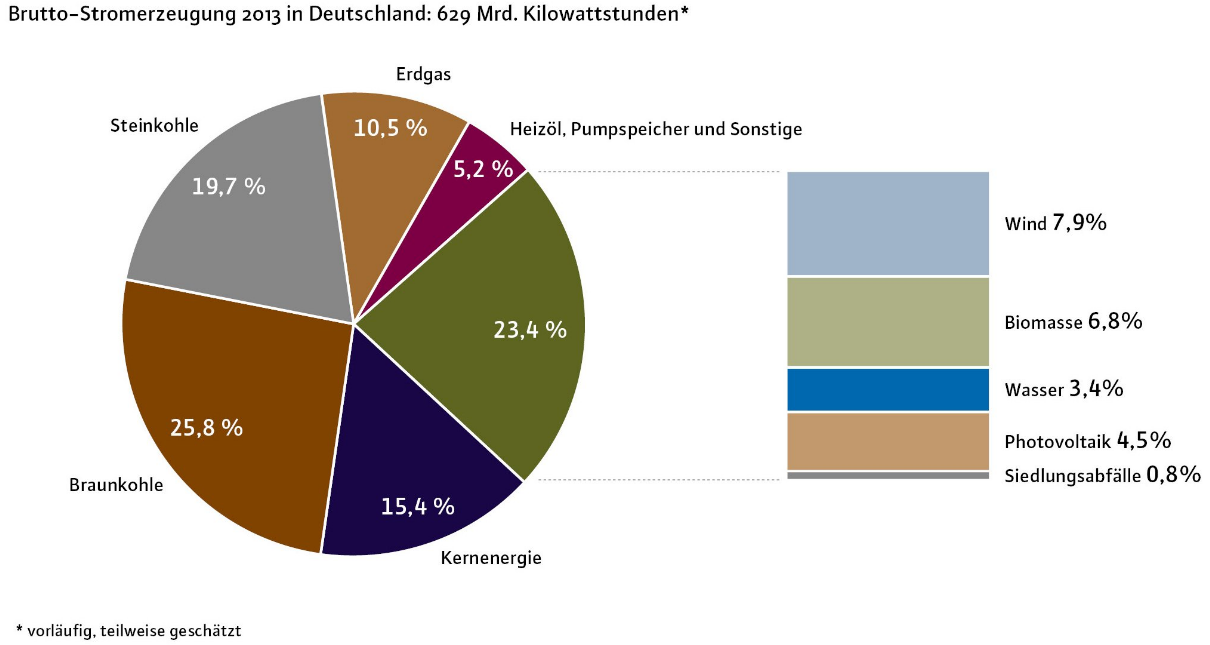


Figure 5.1: German gross electricity generation by energy source in 2013 (25.8% brown coal, 23.4% Renewables, 19.7% black coal, 15.4% nuclear, 10.5% gas, 5.2% other) [7]

or a gas engine is used air pollutants and greenhouse gases are emitted only locally. The most important air pollutants and greenhouse gases are listed in the following:

- Carbon dioxide (CO2) [g]
- Sulfur dioxide (SO2) [g]
- Nitrogen oxide (NOx) [ppm]
- Suspended particulate matter (SPM) [ppm]
- Diesel particulates [ppm]

Depending on the used energy source and the energy consumption the output of air pollutants and greenhouse gases is calculated by equation (5.2).

$$x_{local} = M \cdot x_{normed} \tag{5.2}$$

x_{local}	local air pollution (CO2, SO2, NOx, SPM, Diesel particulates)
M	total amount of energy source (e.g. diesel [l], electricity [kWh])
x_{normed}	air pollution (e.g. CO2) of a normed amount of the energy source (e.g. 1 l of diesel, 1 kWh)

When a mix of energy sources is used for producing electricity, the global air pollution is calculated by equation (5.3). The equation has to be applied for all energy sources of the mix of electricity separately.

$$x_{global} = (M \cdot share) \cdot x_{normed} \tag{5.3}$$

x_{global}	global air pollution (CO2, SO2, NOx, SPM, Diesel particulates)
M	total amount of energy source (e.g. diesel [l], electricity [kWh])
$share$	share of the energy source within the energy mix [%]
x_{normed}	air pollution (e.g. CO2) of a normed amount of the energy source within the mix (e.g. brown coal; amount of brown coal which is necessary for producing 1 kWh of electricity)

5.2 Hazardous substances

Each energy source includes hazardous substances which have impacts on health (Carcinogenic, Mutagenic and toxic to Reproduction (CMR). Regarding European Directive 67/548/EEC [3] and the Regulation (EC) No 1272/2008 [4] which establishes the globally valid GHS (Globally Harmonized System of Classification and Labelling of Chemicals) [10] at European level there are the following relevant categories:

- Category 1: evidence from human experience (GHS: 1A)
- Category 2: demonstrated in animals, is suspected in humans (GHS: 2B)
- Category 3: assumed that it is (GHS: 2)

Within the CACTUS project a statement can be given whether there are emitted hazardous substances or not depending on the energy source.

5.3 Further improvements

Electrically driven vehicles open an additional source for energy by storing the breaking energy. This operation is called *recovery* and can be used in normal traffic situations which require breaking as well as downhill drives. Energy recovery is an option for modern electrically driven buses.

In the CACTUS project we defined the charging policy *charging on ways* (see section 3.3.1) which uses conductive or inductive energy supply during normal bus drives. This operations allows charging batteries and using the offered power for driving the bus directly.

This direct supply of the engines does not need the indirection thru any battery and has an efficiency of about 100%. Furthermore there is no aging of the battery for this amount of energy. Finally the life time of the battery increases.

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