CACTUS
Models and Methods for the Evaluation and the Optimal Application of Battery Charging and Switching Technologies for Electric Busses

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Deliverable 2.1
Models

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1 Introduction

The global trend towards clean and energy-efficient vehicles is driven by concerns regarding the impacts of fossil-fuel-based road transport on energy security, climate change and public health. Electrification in particular is understood as providing a potential multitude of opportunities for the use of energy from renewable sources and for the reduction of local emissions and green house gas emissions like no other. In 2009, the European Commission presented the Green Cars Initiative aimed at encouraging the development and market uptake of clean and energy-efficient vehicles. This strategy will enable the environmental impact of road transport to be reduced and will boost the competitiveness of the automobile industry.

1.1 Problem

The use of public transport is an environmentally friendly way to travel. If more and more passenger cars will be powered by electrical energy in the future, public transport companies will be forced to convert their diesel busses into electric busses in order not to lose this advantage.

The requirements of busses are different to those of passenger cars. A bus covers an average distance of 250 to 300 km each day. The bus itself has a weight of, for example, 14 – 17.5 t (Solaris Urbino 18), 28 t (MAN NG 313) or 26.6 t (Mercedes O 405 GN). A suitable battery that would enable the bus to run for such a long distance without having to be recharged would be far too big, heavy and expensive. In order to overcome this problem, several approaches are currently being investigated, for example switching the battery and the short inductive charging of supercapacitors at bus stops. With these technical solutions, which combine vehicles and infrastructure, fully electric busses should be enabled for use in public transportation.

Assumptions:

1. In the near future, there will be no batteries for fully electric busses which provide the daily output of 300 km without needing to be recharged and which would be acceptable in terms of their size, weight and cost.
2. No technical approach that is currently being investigated will be equally suitable for all public transport companies.
3. In any case, investment costs for vehicles, in-vehicle components and infrastructures (e. g. battery charging or battery switching facilities) will be very high for public transport companies.

The following conclusion is drawn: The available technical approaches and solutions must be considered separately against the prerequisites and requirements of every single public transport company in terms of transportation, technical, economic and environmental aspects. Only on this basis can a decision for a technology that optimally meets the requirements of a public transport company be made.

1.2 Objectives

Technical solutions to enable fully electric busses should be evaluated so that they reflect the prerequisites and requirements of the participating public transport companies. The ultimate goal of the project is to find the best technical solution for the participating public transport companies HVB, MVB, PVGS and PKM depending on their real input data (timetable, vehicle operation plan, etc.), which in most cases may mean minimising the investment and operational costs. Of course, the best solution may vary between the participating public transport companies due to the strongly different prerequisites, assignments and aims. The best solution does not only involve a technology, but also its optimal application.

To achieve this aim, models of all relevant transportation, technical, economic and ecological values will be elaborated. Methods will be developed with which the question as to the most suitable technical solution (depending on the input values) can be answered and which help to apply the technical solution found in an optimal way. A software tool will be developed with which the different solutions can be easily compared. It should be possible to study the gradual integration of fully electric busses into existing fleets of diesel, natural gas and hybrid busses.

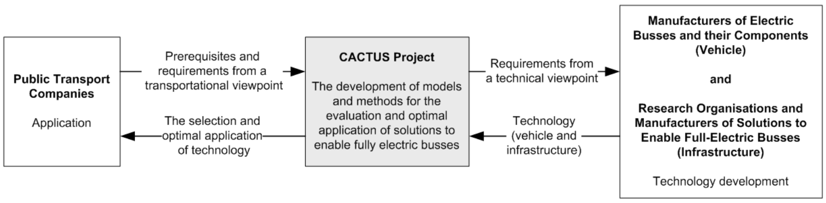

Figure 1: The role of the CACTUS project

The preliminary studies with the participating public transport companies will be lead into recommendations for the actors in the field of technology development, namely the manufactures and researchers of fully electric buses and the corresponding infrastructure. The role of the CACTUS project can be seen in Figure 37.

1.3 Concept

In the CACTUS project, considerations concerning techniques for fully electric buses will be made to decide which best fits a public transport company's needs. This requires a series of detailed questions to be answered. Some general questions are:

- Is it possible to keep to the timetable with a given configuration (all technical and strategic elements requiring the operation of fully electric busses), a given vehicle fleet (including those with mixed engines) and a given vehicle operation plan?
- How high are the investment and operational costs?

In this context, several optimization issues arise, some of which are listed here:

- What should the operation plan look like so the timetable can be kept to?
- Where the charging or exchanging facilities have to be located?

In the CACTUS project, methods that can be used to answer these will be developed.

1.4 Scope of this Deliverable

On the basis of collected issues the necessary input variables will be identified and the output variables will be specified. The output variables allow a quantitative assessment and comparability to transportation, technical, economic and ecological aspects. If required input variables are not yet available in electronic form will be collected and prepared, so that they can be electronically processed. All variables and their relations to each other will be modeled, so the methods to be developed in WP3 can work with.

In Deliverable 1.1 the questions have been collected which will be answered within the CACTUS project. The Deliverable 1.1 only mentioned *what is the problem* not *how will the problem be solved* (this is part of Deliverable 2.1 and moreover Deliverable 3.1).

Deliverable 1.2 provides a large collection of current technologies for enabling fully electric buses as well as a broad overview to current available electric buses. The technologies includes lithium-ion batteries, inductive charging, charging via pantograph on the run, exchanging the battery and the application of super capacitors.

The models presented in this Deliverable 2.1 are simplified images of the reality. They are divided into transportation, technical, economic and ecological complexes. Some will be considered as input values and some as output values. A method to be developed in WP3 works with input values and delivers output values. Models are the basis of the methods to be developed in WP3. The models are a pre-stage of implementation. That means it is aimed to write the models in source code.

2 Modeling Methods

Different types of modeling methods are applied within this Deliverable. The selection depends on the subject area. Other types of modeling methods are used for economic models and ecological models.

2.1 Object Oriented Modeling

The method of Object Oriented Modeling (OOM) together with the method of Object Oriented Analysis (OOA) is used for transportation and technical aspects. The Unified Modeling Language (UML) is applied in order to illustrate the models. The diagram type Class Diagram is used. The class diagrams do not have operations at the time (only attributes), because the models only represent data without any operations. Operations will be added during the implementation and the development of methods (see Deliverable 3.1.1). Within the class diagrams, no difference between composition relation and aggregation relation is made because this is not relevant for implementation.

2.2 System Theoretic Models

In the CACTUS project models are abstract and simplified mathematical descriptions for the real world behavior of transport systems based on electrically driven buses. These models are used to estimate the level of charge of all batteries in the system. Finally they are needed to localize and dimension the charging infrastructure.

We assume a linear behavior of all components. This is only partly acceptable and it does not hold in general. The influence of temperature on battery capacity and the necessary energy are just examples. We define those input values as *parameters* which change slowly over the time and which can be assumed as constant even under worse case conditions over a long period of time.

It is reasonable to run this simple approach in order to keep the system wide model slim and find solutions quickly. The results should give start values for a subsidiary simulation run which can verify or improve the solution.

In section 4.1 to section 4.2 we develop models for all components. In section 4.3 system wide models for cyclic operations are introduced. This is a very common operation mode in urban areas. For other cases a more detailed model is given in section 4.4.

The introduced models are presented as block diagrams and in some cases as equations. Every model has input variables, parameters and output variables. Input variables are indicated by 1 while output variables are indicated by 2. Parameters are not indicated in general but initial value parameters are indexed by zero. Input and output variables are functions of time while parameters are constant. In some cases variables are written as vectors (small and bold letters) or matrices (capital and bold letters). [1]

These Models are developed with respect to analytical aspects and numerical computation. For this reason the physically continuous signals are treated as discrete value series. This can be done in two ways:

Sample System All values are measured and calculated in equidistant values of time, the sampling time T.

Event System All values are measured and calculated when an event occurs, e.g. a bus arrives at a bus stop or it leaves a bus stop.

A sample system can be expressed in the time domain by functions of $kT \quad k \in \mathbb{N}$ and in the z domain by functions of z.[2] A typical example is the integrator which is typical to calculate the energy $e(t)$ from a given power $p(t)$ by integration (see equation (1)). In a sample based system this can be done by a step function with a step width of T (see equation (2)). A simplified notation omitting T is shown in equation (3).

$$e(t) = \int p(t)dt \tag{1}$$

$$e(kT) = T \cdot p(kT) + e((k-1)T) \qquad k \in \mathbb{N} \tag{2}$$

$$e(k) = T \cdot p(k) + e(k-1) \qquad k \in \mathbb{N} \tag{3}$$

The signal flow of sampled systems can be shown by block diagrams. The graphical symbol for a multiplication of a signal with a constant is shown in figure 2a. The delay of a signal is shown in figure 2.

Figure 3 shows a time distance diagram for 3 bus stops connected by 2 paths. The slope represents the velocity of the bus.

Figure 4 shows the power p_B for the base load like air-condition and headlights added with the power p_D for the drive load displayed in a single graph. The bus is switched on at $t = t_1$. At this time the basic load is active and the bus starts driving immediately. The first stop is between t_2 and t_3 and the final stop starts at t_4. At t_5 the bus is shut down and power consumption drops to zero.

All values can be measured and processed by events or by samples.

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[1] This is the reason why we use p and e for power and energy rather than P and E which are common in physics. The only exception from this rule is E_0 which is used as a constant value for the initial Energy.

[2] The z-domain is not further used in the document but it could be useful later in the project.

(a) Multiplication with a constant factor a.

(b) Delay by a period of T seconds.

Figure 2: Principles of sample based and event based systems.

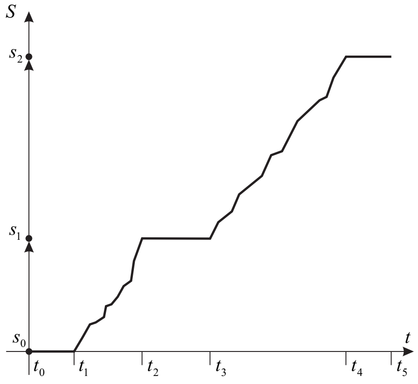


Figure 3: Time distance diagram for 3 bus stops connected by 2 paths.

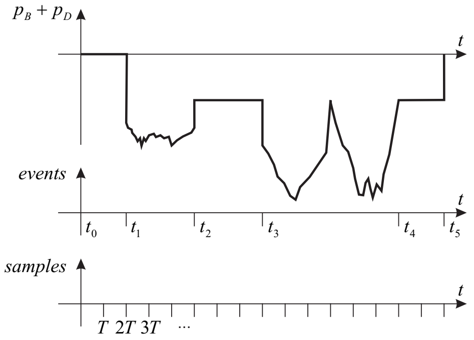
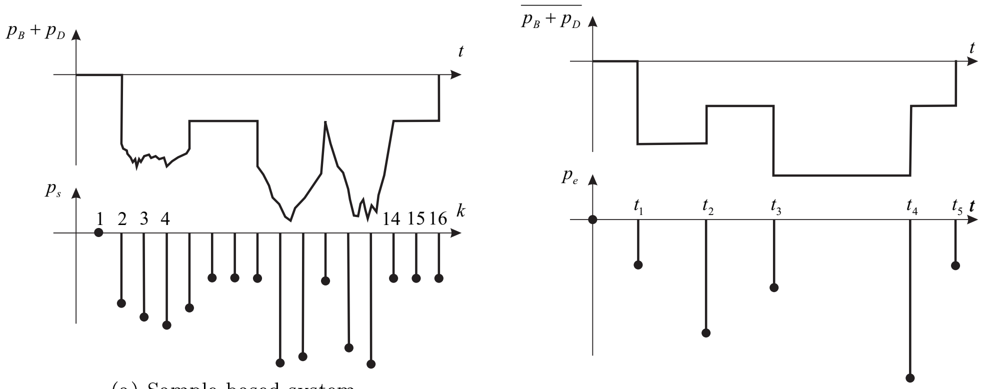


Figure 4: Power consumption is the sum of basic load p_B and drive load p_D. The values are measured and processed at event times t_i or at sample times kT where T is the sample time.

In a sample driven system the sample represent the current values at the sample time kT (see figure 5a). Event driven systems use the mean values between the last and the current sample multiplied by the time between these samples. This is the area of that part of the graph (see figure 5b).



(a) Sample based system

(b) Event based system

Figure 5: Principles of sample based and event based systems.

3 Objective Oriented Models

3.1 Transportation Models

The transportation input and output variables will be identified and modeled within this chapter. Among others, these include the road network, the transport network, the timetable, the operation plan, the topography, speed profile (e. g. stops caused by traffic signals) and the ridership.

3.1.1 Transport Network

The network of the bus routes is the basic. It includes a set of bus stops. Each bus stop is described by its id, its name and its geographical location. Bus stations may have several platforms (usually at least two - one for each direction). Each stop exactly represents such a platform. Figure 38 shows the UML class diagram of a bus stop. Here not only bus stops for passengers have to be considered but depots as well. Because for battery recharging all locations are interesting where a bus stays.

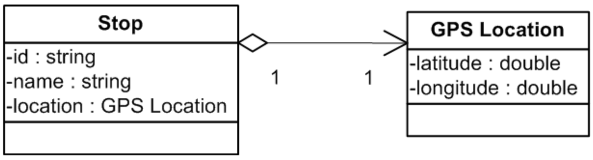


Figure 6: Model of a bus stop

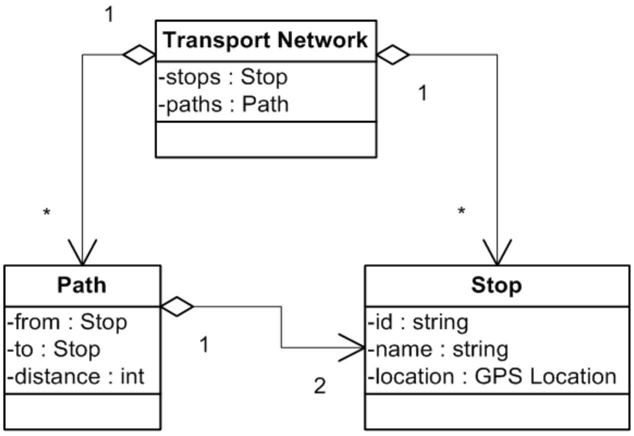


Figure 7: Model of a path and the transport network

A path directly connects two bus stops. Such a path is characterised by its distance in meters and the connecting stops (attributes *from* and *to*). The distance means the length of the way the bus needs to cover to get from the *from* stop to the *to* stop. It is not the beeline. The transport network holds a list of stops and a list of paths (see Figure 7).

3.1.2 Timetable

The timetable simply is a set of runs. A run is the journey of a bus from a start stop via a set of stops to an end stop. Its attributes are the id, the route number, the bus type and the travel dates. The ordered list of the bus stops is modeled as a list of moves (class *Move*) whereas each move references to a path. Furthermore, a move stores the departure time at the *from* stop and the arrival time at the *to* stop of the corresponding path. In public transport the arrival time mostly is equal to the departure time at the same stop. Figure 8 provides the UML class diagram of the timetable.

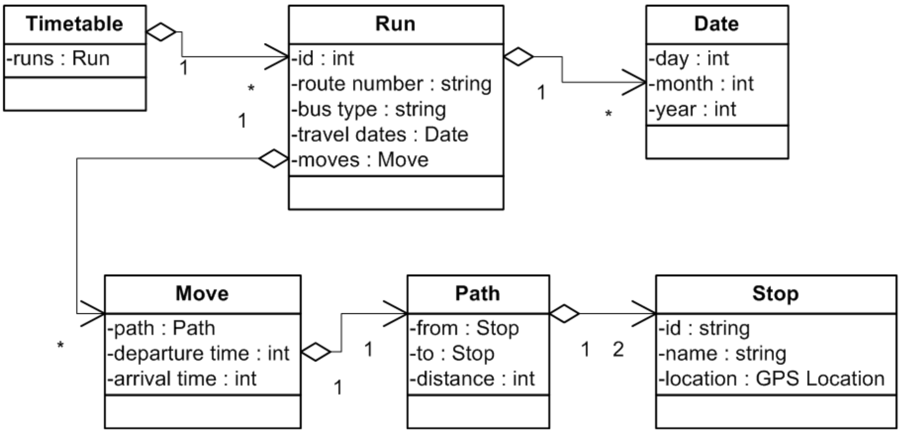


Figure 8: Model of the timetable

The departure time and the arrival time is considered as minute of the day in the range of $[0, 1439]$.

3.1.3 Road Network

Buses run on roads. Within the CACTUS project it is important to know the roads where the buses run in order to identify suitable locations for charging infrastructure. Within the CACTUS project the map database OpenStreetMap (OSM) is used because it is free.

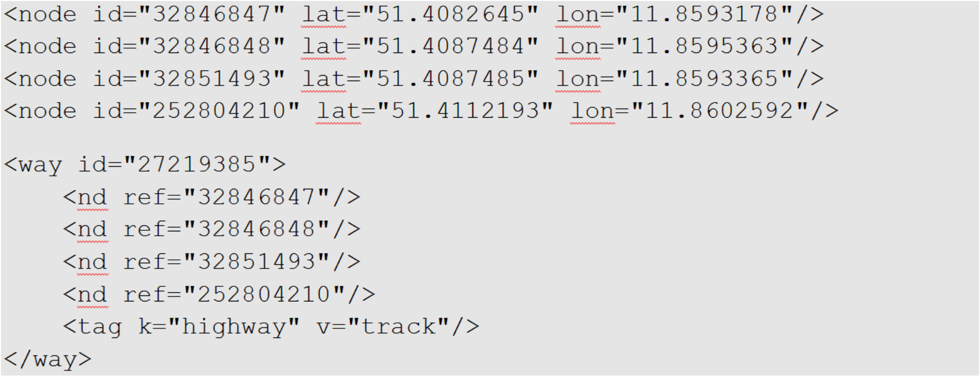
OpenStreetMap files are XML based. The most important elements within OpenStreetMap are *Node* and *Way*. A way is composed of a list of nodes (tag <*nd*>). Figure 11 provides a very short sample part of an OpenStreetMap file.

A path between two bus stops can be considered as an OSM way. It is also composed of a list of OSM nodes. Figure 10 depicts the class diagram of the path extended by a list of OSM nodes.

OpenStreetMap even provides the special tag <*relation*> which can be used for public transport routes. Unfortunately, the OpenStreetMap database does not contain all routes of the public transport companies participating in the CACTUS project.

3.1.4 Topography

The topography is a description of the natural earth surface with heights and depths. For conventional buses with diesel, gas or hybrid-engine this is not relevant. But pure



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<node id="32846847" lat="51.4082645" lon="11.8593178"/>
<node id="32846848" lat="51.4087484" lon="11.8595363"/>
<node id="32851493" lat="51.4087485" lon="11.8593365"/>
<node id="252804210" lat="51.4112193" lon="11.8602592"/>

<way id="27219385">
    <nd ref="32846847"/>
    <nd ref="32846848"/>
    <nd ref="32851493"/>
    <nd ref="252804210"/>
    <tag k="highway" v="track"/>
</way>
```

Figure 9: Simplified sample part of an OpenStreetMap file

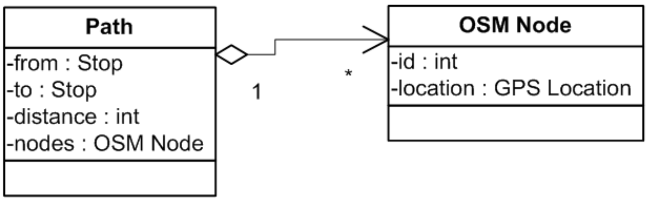

Figure 10: Model of the road network

electric buses are limited to its range, so this must be considered as well. Figure 11 provides a simple sketch of a fictive topography. Each value is characterised by its location in GPS coordinates and its height in meters above sea level.

The model of the topography is depicted in Figure 12. The topography is composed of a list of topography entries whereas each instance of *Topography Entry* class holds a location and a height value. Each instance of the *Topography* class belongs to a path. The *Path* class has been extended by the topography.

3.1.5 Speed Profile

The speed profile of a path is statically modeled here. For conventional buses with diesel, gas or hybrid-engine this is not relevant. But pure electric buses are limited to its range, so this must be considered as well.

Figure 13 depicts a simple sketch of a fictive speed profile. It starts with the start bus stop and ends at the destination bus stop at a speed of zero (that means still standing so that passengers can leave or enter the bus). It is obviously, that the speed profile of a path is never the same at every time. Halting or not halting at a signal controlled intersection depends of the signalling method (e.g. precemption of public transport vehicles, immutable signalling cycle), the random arrival of the bus and the current time within the signalling cycle. Left turns sometimes require a total stop and sometimes not. But we suppose that there are similarities between all these different real speed profiles. It even might occur that the bus does not halt at the bus stops because the bus only stops if passengers waiting at the bus stop for entering the bus or if passengers want to exit (they have to announce this by pressing a button within the bus).

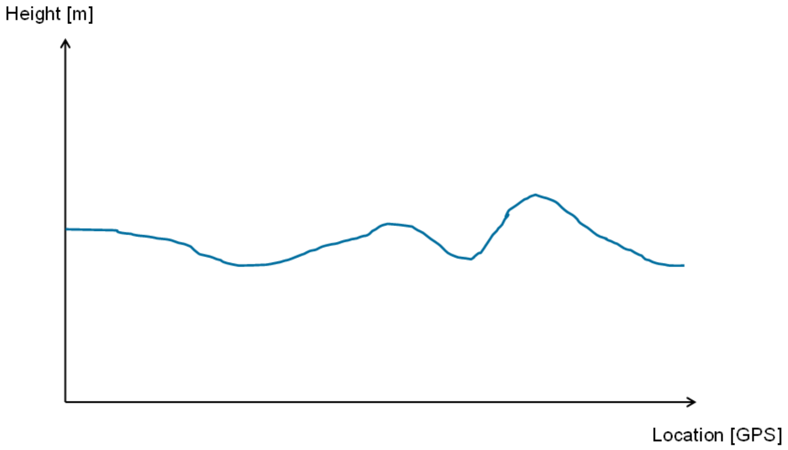

Figure 11: Simple sketch of a fictive topography

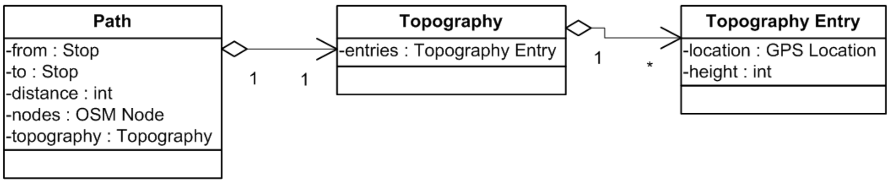

Figure 12: Class diagram of the topography

Figure 17 provides the speed profile of a real course in Brunswick, Germany, with a length of 11 km [2]. Compared to Figure 16 the x-axis has the unit seconds instead of location.

Figure 15 depicts the UML class diagram of the *Path* class extended by the speed profile.

3.1.6 Ridership

Ridership means how many passengers are transported on each run between which bus stops. For conventional buses with diesel, gas or hybrid-engine this is relevant for planning the timetable and the vehicle sizes. This is also relevant for pure electric buses. But pure electric buses additionally are limited to its range. The energy consumption and therefore the range of pure electric buses depend on the weight of the bus which is influenced by the number of passengers. That is why the number of passengers must be considered as well. The sketch in Figure 18 shows a fictive passenger profile. The bus stops are plotted on the x-axis. The numbers of passengers are plotted on the y-axis.

Figure 18 provides the model of the ridership. The *Move* class (introduced in 3.1.2) has been extended by the attribute *number of passengers*.

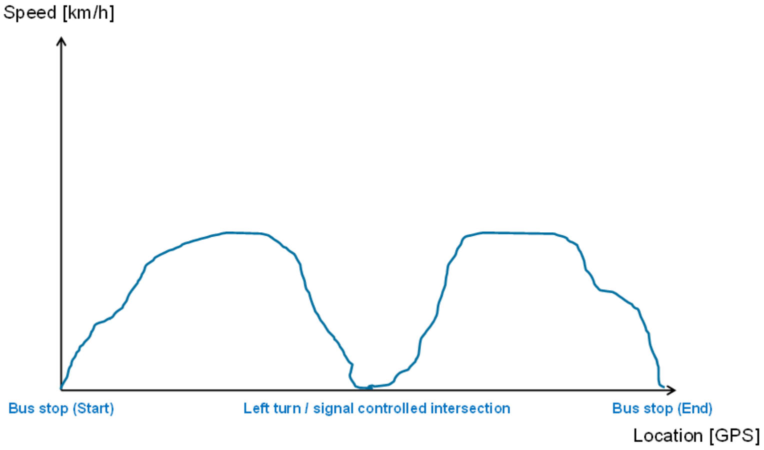

Figure 13: Simple sketch of a fictive speed profile of a path

Activity	Start Time	End Time	Start Location	End Location
Prepare	11:17	11:37	Depot	Depot
Refuel	11:37	11:52	Depot	Depot
Move Out	11:52	11:53	Depot	Main Station
Run 32746	11:53	14:30	Main Station	Market
Move In	14:30	14:35	Market	Parking Site
Break	14:35	15:05	Parking Site	Parking Site
Move Out	15:05	15:10	Parking Site	East End
Run 3462	15:10	16:53	East End	Main Station
Move In	16:53	17:13	Main Station	Depot
Finish	17:13	17:43	Depot	Depot

Table 1: Sample of a fictive work schedule

3.1.7 Operation Plan

The operation plan holds the schedules of the concrete vehicles. It is derived from the work schedule. Table 1 provides a sample of a fictive work schedule of a bus driver which is leaned on a real one. The work schedule determines an ordered list of instructions the bus driver has to process. Each activity is described by a name, the start and the end time as well as the start and the end location. The most important activities are the runs but it also reveals longer standing times which could be opportunities for recharging the battery. Furthermore, the work schedule is valid only on certain dates.

Figure 21 provides the model of the operation plan. The class *Operation Plan* holds the references to all work schedules.

The *Work Schedule* class has an Identifier (attribute *id*) and holds references to a bus and to a list of activities (class *Operation Activity*). Runs (see Section 3.1.2) are considered as special activities.

3.2 Technical Models

The technical input and output variables will be identified and modeled. Among others, these include vehicle characteristics and behavior, resultant energy consumption,

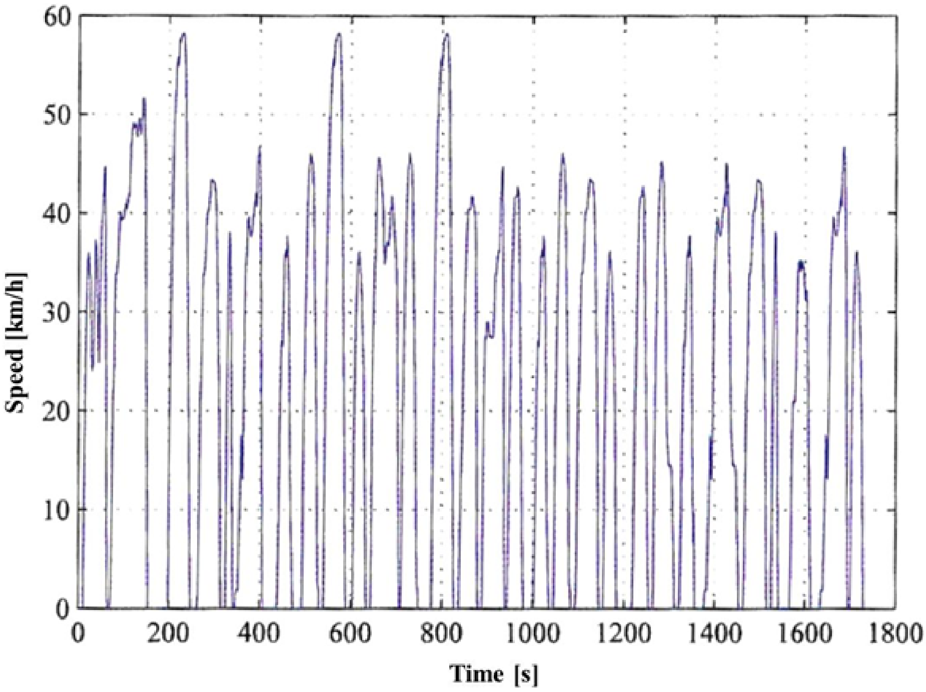

Figure 14: Speed profile in Brunswick [2]

battery characteristics and behavior, temperature, battery replacement or recharging strategy in operational mode. Diesel buses, natural gas buses and hybrid buses will be also modeled in order to investigate mixed fleet and the gradual introduction fully electric buses.

3.2.1 Bus

General A bus in general has an empty weight. This is the mass of the bus without a battery and without any passengers. The bus has a number of maximum standing places and seats. These attributes belong to the abstract class *Bus* (see Figure 22). For diesel buses, natural gas buses and hybrid buses the concrete class *Conventional Bus* is provided. The type of the bus *DIESEL / GAS / HYBRID* is assigned by the attribute *type*.

This model of the electric bus will be extended during the next sections.

Regenerative Brake The regenerative brake is a mechanism for energy recovery which slows the bus down by converting its kinetic energy into electrical energy, which can be either used immediately or stored into the battery. The most common form of a regenerative brake uses an electric motor as an electric generator.

Figure 23 provides the reusable braking energy depending from the power of the generator [2].

The curve of the energy to be recovered depends on the total weight of the bus (including passengers and battery), the initial velocity of the car before braking and the amount of time the bus is braking. Figure 24 provides the model of the extended

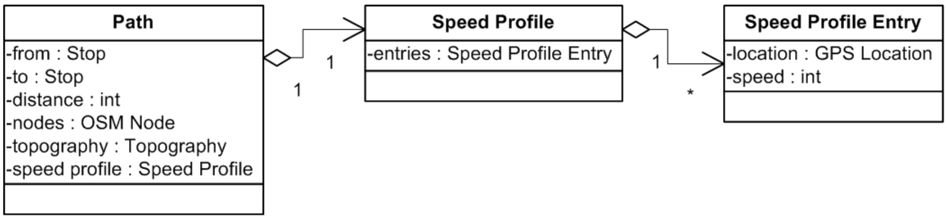

Figure 15: Model of the speed profile

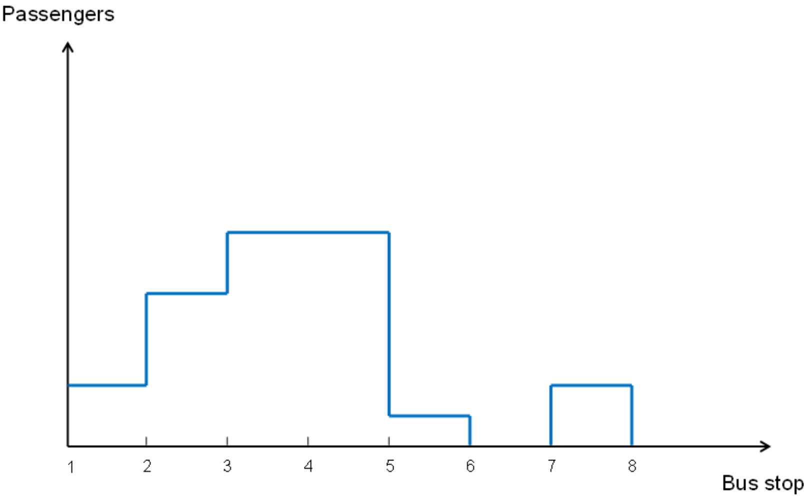

Figure 16: Simple sketch of a fictive passenger profile

by the regenerative brake. A bus may be equipped with such a regenerative brake or not.

Electrical Consumers A bus has a lot of electrical consumers (excluding the drive) which consume electrical energy from the battery too. Depending on external conditions (daytime, temperature, weather) they consume more or less electrical energy. The demand of these electrical consumers might significantly reduce the range of the bus.

Figure 23 depicts a simple sketch of the supposed energy consumption trace depending on the outside temperature. That considers the air climate system. At about 20°C-22°C, no heating and no cooling is necessary. The lower the outside temperature, the more energy is consumed for heating. The higher the outside temperature, the more energy is consumed for cooling. Please note that this curve shape is only a simple sketch. It does not reflect real values, because this is not necessary for building a model. Real values will be gathered later from the participating public transport companies. This regards specifically policies of the public transport company, e.g. how comfortable the air climate system is adjusted.

Figure 22 depicts the model the bus extended by the model of electrical consumers. A consumer of electrical energy is characterized by its name and by its energy consumption in W/h.

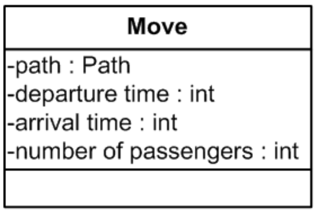

Figure 17: Class diagram of the ridership

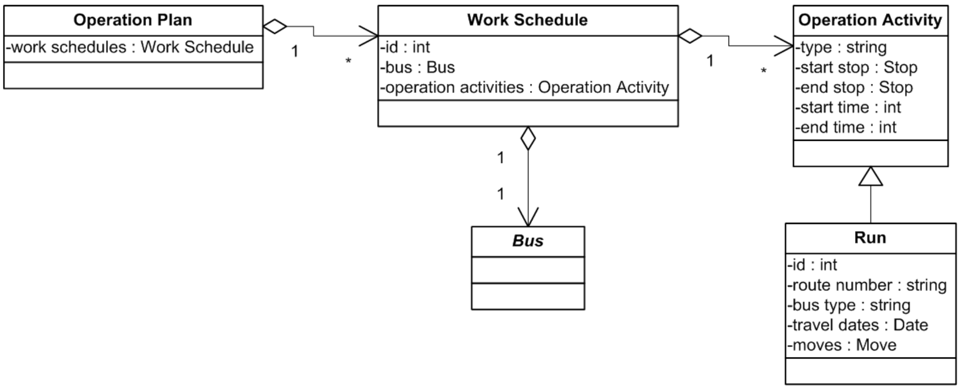

Figure 18: Class diagram of the operation plan

Chassis
The chassis of the bus includes the following components consuming electrical energy:

- Anti-lock braking system (ABS)
- Electronic braking system (EBS)
- Electronic stability control (ESC)
- Power steering
- Niveau regulation system
- Motor management
- Braking assistant
- Engine cooling
- Windshield wiper

Individual components may be added.

Lighting System
The chassis of the bus includes the following components consuming electrical energy:

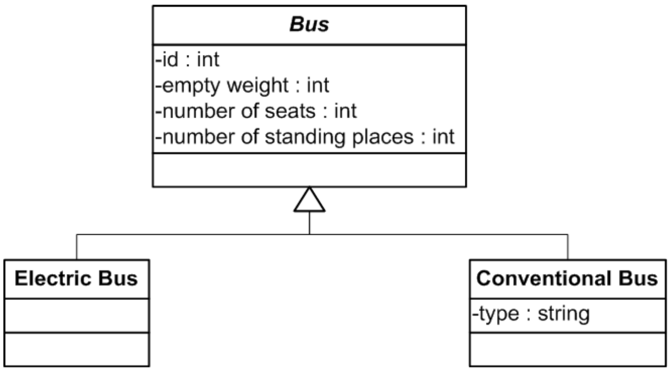

Figure 19: Model of the bus

- Main beam
- Dipped beam
- Turn signals
- Driving lamps
- Fog lamps (front and rear)
- Daytime running lamp
- Registration plate lamp
- Position lamps (front and rear)
- Stop lamps
- Reversing lamps
- Side marker lights
- Passenger compartment lights

Individual components may be added.

Air Condition System
The air condition system of the bus includes the following components consuming electrical energy:

- Heating
- Ventilation
- Air condition

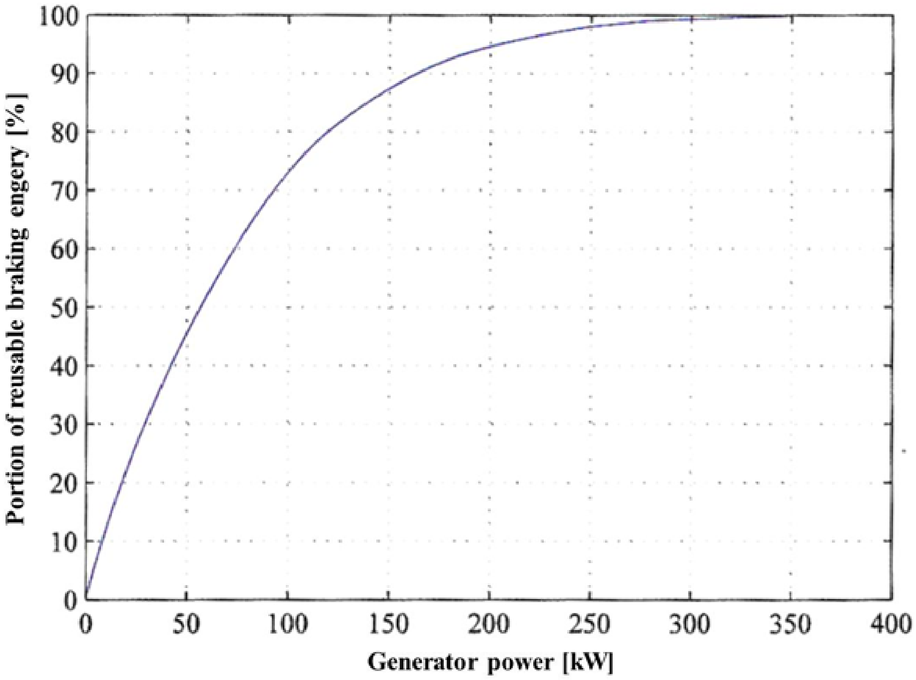

Figure 20: Reusable braking energy depending from the generator power [2]

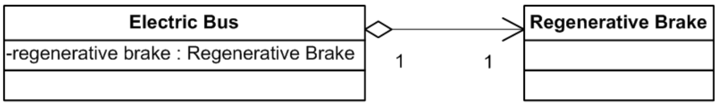

Figure 21: Model of the bus extended by the regenerative brake

Individual components may be added. It is supposed that these components consume the major part of electrical energy from all electrical consumers (excepting the drive).

Passenger Service System
The passenger service system of the bus includes the following components consuming electrical energy:

- Bord display
- Camera surveillance system
- Route display
- Speakers
- Door system
- Passenger button system

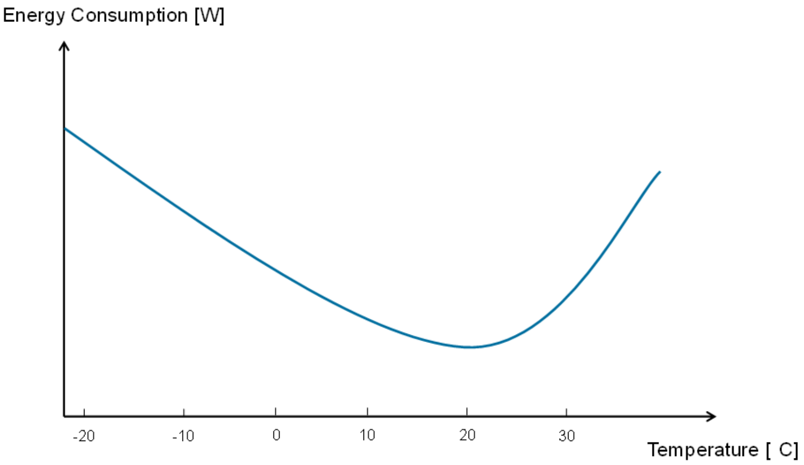

Figure 22: Supposed energy consumption of the bus depending on the outside temperature

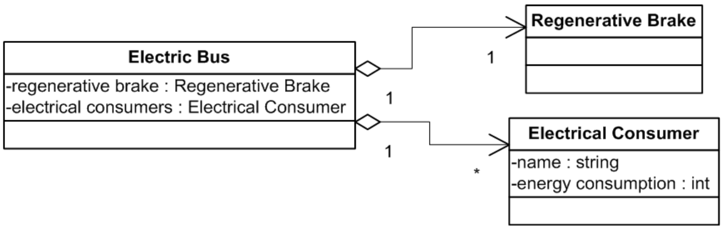

Figure 23: Model of the bus extended by the list of electrical consumers

- Ticket machine

Individual components may be added.

Energy Consumption Figure 24 sketches the forces acting on a car [1].

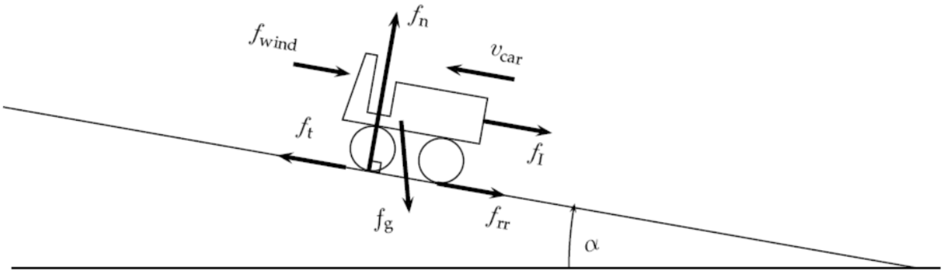

Figure 24: Forces acting on a car [1]

The traction force of a vehicle can be described by the following two equations [1]:

$$\begin{aligned} F_t = M_{car}\dot{v}_{car} + M_{car} \cdot g \cdot sin\alpha + sign(v_{car})M_{car} \cdot g \cdot cos\alpha \cdot c_{rr} + \\ sign(v_{car} + V_{wind})\frac{1}{2}p_{air}C_{drag}A_{font}(v_{car} + V_{wind})^2 \end{aligned} \tag{4}$$

$$c_{rr} = 0.01(1 + \frac{3.6}{100}v_{car}) \tag{5}$$

where

F_t	$[N]$	Traction Force of the vehicle
f_I	$[N]$	Inertial force of the vehicle
f_{rr}	$[N]$	Rolling resistance force of the vehicle
f_g	$[N]$	Gravitational force of the vehicle
f_n	$[N]$	Normal force of the vehicle
f_{wind}	$[N]$	Force due to wind resistance
α	$[rad]$	Angle of the driving surface
M_{car}	$[kg]$	Mass of the vehicle
v_{car}	$[m/s]$	Velocity of the vehicle
$\dot{v}_{car}$	$[m/s^2]$	Acceleration of the vehicle
$g = 9.81$	$[m/s^2]$	Free fall acceleration
$p_{air} = 1.2041$	$[kg/m^3]$	Air density of dry air at 20°C
c_{rr}	$[-]$	Tire rolling resistance coefficient
C_{drag}	$[-]$	Aerodynamic drag coefficient
A_{front}	$[m^2]$	Front area
v_{wind}	$[m/s]$	Headwind speed

The equations (4) and (5) are a good starting point for calculating the energy consumption of a bus depending on the mass, the topography and the speed profile.

3.2.2 Battery

A battery is not strictly bound to a bus because they may be exchanged for a battery of the exchanging station. A battery has the following attributes:

- *id:* The unique identifier of the battery within model.

- *manufacturer:* The name of the manufacturer of the battery.
- *type:* The type of the battery. This helps to secure that only compatible batteries will be charged or exchanged.
- *weight:* The weight of the battery in kg.
- *capacity:* The capacity of the battery in kWh.
- *age:* The age of the battery in months.
- *number of full charging cycles:* The number of full charging cycles the battery has passed through.
- *charging level:* The actual charging level of the battery.
- *maximal charging current:* The maximal current with which the battery can be recharged.

In Figure 27 the model of the bus has been extended by the model of the battery. A bus may have not only one battery but more of them.

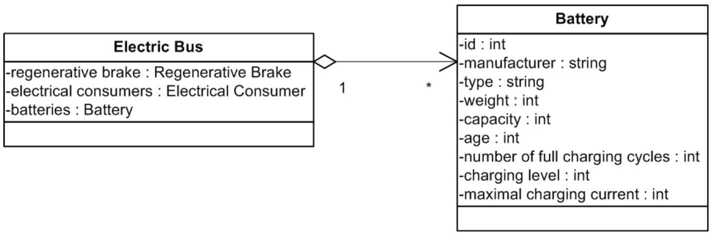

Figure 25: Model of the bus extended by the battery

3.2.3 Energy Sources

There are several energy sources for fully electric buses. This means the bus obtains electrical energy. Known are charging stations, charging tracks and exchanging stations. At charging stations, the bus has to stop for obtaining the electrical energy. The energy transfer can be performed via cable or inductively (without cable). At charging tracks, the energy can be transferred while the bus is running. This may work inductively or via a pantograph. Exchanging stations hosts one or more batteries. When a battery is exchanged, the battery is taken out of the bus and put into the exchanging station for recharging. After that a recharged battery is taken out of the exchanging station and put into the bus. Empty batteries taken out of the bus will be recharged within the exchange station then. Energy sources are connected to the road and transport network by their location.

Figure 22 depicts the model of the possible energy sources: the abstract class *Energy Source* is specialized by the classes *Charging Station*, *Exchanging Station* and *Charging Track*.

The common attributes of all energy sources are:

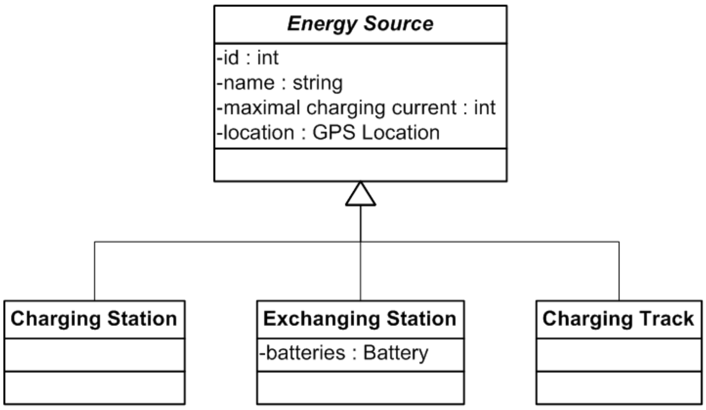

Figure 26: Model of charging stations, charging tracks and exchanging stations

- *id:* The unique identifier of the energy source.
- *name:* The name of the energy source.
- *maximal charging current:* The maximal current with which batteries can be recharged.
- *location:* The GPS location of the energy source.

Exchanging stations additionally own a depot of one or more batteries (attribute batteries).

3.2.4 Energy Regeneration Plan

The energy regeneration plan is a list of activities for supplying the bus with electrical energy. This can be done by different sources as explained in section 3.2.3. All activities are inherited from the class *Operation Activity.* The following attributes from the superclass are used:

- *start time:* The start time of the activity. For charging activities, this is time the charging process starts. For exchanging activities, this is the time the exchanging process starts.
- *end time:* The end time of the activity. For charging activities, this is time the charging process ends. For exchanging activities, this is the time the exchanging process ends.

Additionally, charging activities have the following attributes:

- charging current: The charging current, when charging is performed.
- energy source: This is a station or a track, where the recharging of the battery is performed.

Finally, each exchanging activity is connected to an exchanging station (attribute *exchanging station*). This is the station where the battery of the bus is exchanged to a battery of the exchanging station.

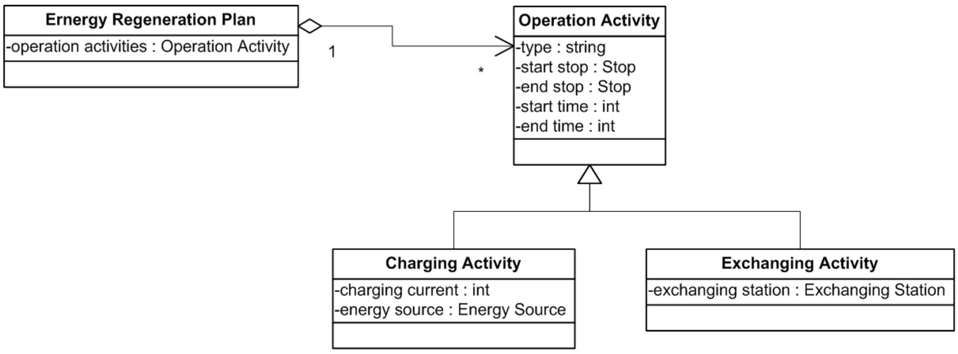

Figure 27: Model of the energy regeneration plan

The energy regeneration plan can be an input variable for simulation as well as an output variable of optimisation. The operation plan (see section 3.1.7) is complemented by these activities.

4 System Theoretic Models

4.1 Technical Models

Technical models deal with batteries, power and energy. They define some of the basics for the system wide models introduced in sections 4.3 and 4.4.

4.1.1 Battery model

The battery model should be applicable to all types of batteries which are currently used as well as to such types of batteries which will be available in the future. That means the model must be parameterizable to meet different battery types. Battery models mostly have the battery voltage as output value . This is not helpful in the CACTUS project because we only need the available energy independently from the type of battery.

We assume a parameterizable linear model. All nonlinear influences can be defined as parameters which will be assigned to fixed values before we make use of the model. If we regard a special operating point we get a linear model which is sufficient for analyzing some properties and for synthesizing the charging infrastructure.

Energy model The input variable of the energy battery model is either the energy or the power p which are positive while charging and negative while driving. The output e_2 is the energy currently available from the battery (see figure 28).

$$\begin{aligned} e_i(k) =& T \cdot p_1(k) + e_1(k) + e_i(k-1) && (6) \\ e_2(k) =& e_i(k) + E_0 && (7) \end{aligned}$$

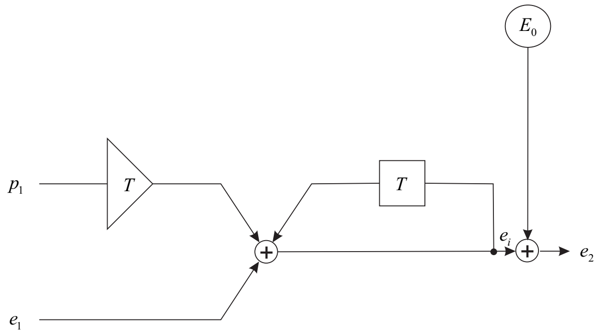

Figure 28: Very simple battery model showing the signal flow rather than the energy flow. e_1 and p_1 are the input variables which are positive while charging and negative while discharging. e_2 is the available energy which must be kept in the range $0 \leq e_2 \leq cap$ externally. The initial energy E_0 is a constant parameter value. See equations (6) and (7)

.

In time domain applications it could be useful to work with power as input while event driven applications energy as input is very simple. In the second case we do not need to integrate and we do not need any event time. In some cases both views are needed simultaneously: For a given section of a well known road the energy consumption can be estimated while the base load for air-condition and lightning is given in terms of power. Fortunately the power can be assumed as constant over time and the integration is to be done simply by multiplication the power p with the period of time for which the power is consumed. In the case of battery swapping the energy view is used. The following numerical example shows an empty battery which gets at $t = 3s$ a battery which is charged with 80 Ws. Subsequently a power of 2 Watt is consumed over three sample intervals followed by a charge of 6 Watts for two sample times and than a consumption of 2 Watts.

k=(	1,	2,	3,	4,	5,	6,	7,	8,	9, ...)	
kT=(	1.5,	3,	4.5,	6,	7.5,	9,	10.5,	12,	13.5, ...)	T=1.5s
p_1=(	0,	0,	0,	-2,	-2,	-2,	6,	6,	-2, ...)	[W]
e_1=(	0,	80,	0,	0,	0,	0,	0,	0,	0, ...)	[Ws]
e_2=(	0,	80,	80,	77,	74,	71,	80,	89,	86, ...)	[Ws]

In the next step we consider the efficiency η to express that the input energy e_1 is only a part of the input energy is stored in the battery and the output does not deliver all the energy which is stored inside the battery. We introduce the charging efficiency η_1 and the discharging efficiency η_2 shown in figure 29.

$$e_i(k) = T \cdot p_1(k) + e_1(k) \tag{8}$$

$$e_2(k) = \big(pos(e_i(k)) \cdot \eta_1 + neg(e_i(k)) \cdot {\eta_2}^{-1}\big) \cdot \eta_2 + e_2(k-1) \tag{9}$$

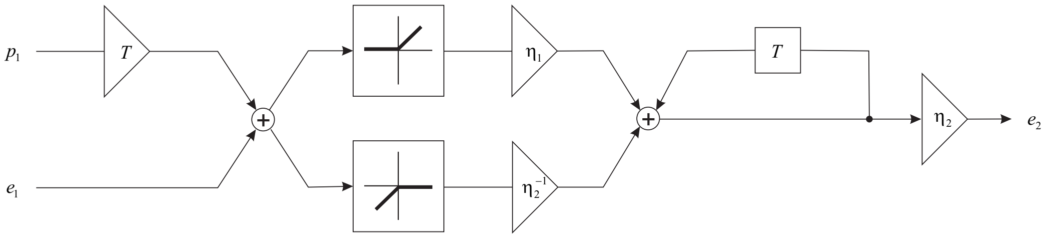

Figure 29: Battery model with limited efficiency η_1 for charging and η_2 for discharging. The input is split in positive values for charging and in negative values for discharging. See equations (8) and (9).

The discharging efficiency η_2 can be multiplied with the charging efficiency η_1 and we get the overall efficiency $\eta = \eta_1 \cdot \eta_2$ at the input and 100% efficiency at the output. This model is shown in figure 30 and will be used in CACTUS project if efficiency should be taken into account.

$$e_2(k) = pos(e_i(k)) \cdot \eta_1 + neg(e_i(k)) + e_2(k-1) \tag{10}$$

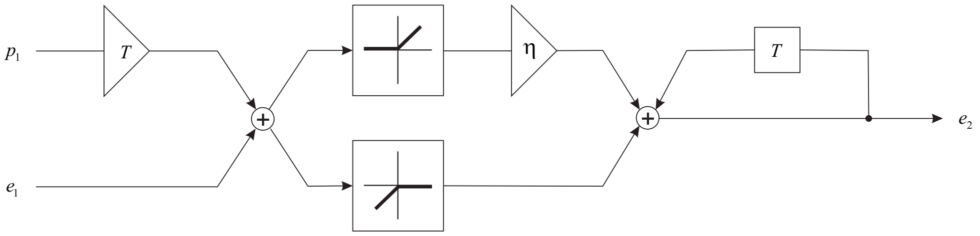

Figure 30: Battery model with limited efficiency $\eta = \eta_1 \cdot \eta_2$. See equations (8) and (10)

This very basic model is now extended by a factor which depends nonlinear on the temperature $Temp$ (see figure 31). Since we compute the function f_{Temp} only once and assume a fixed temperature for the run-time of the model. If this assumption does not hold we have to make additional runs with different values for $Temp$.

$$e(k) = (T \cdot p(k) + e_1(k)) \cdot f_{temp}(Temp) + e(k-1) \quad k \in \mathbb{N} \tag{11}$$

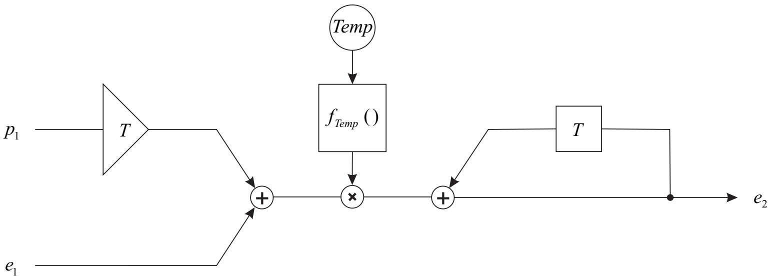

Figure 31: Battery model with nonlinear temperature dependency. $Temp$ is constant and f_{Temp} must be calculated once before the model is used.

The self-discharging effect is expressed by a damping factor a which is in the integration loop (see figure 32). The effect of self-dischargement is to take into account in a long run only. The available energy e_2 can be estimated by the recursive equation

$$e(k) = T \cdot p(k) + e_1(k) + a \cdot e(k-1) \quad k \in \mathbb{N} \tag{12}$$

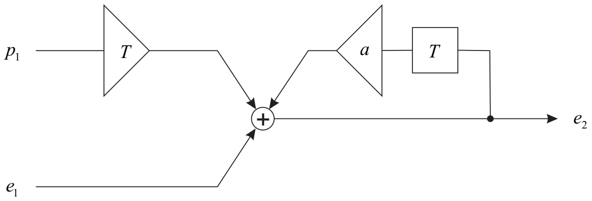

Figure 32: Battery model with self dischargement.

The application of all these simple models must ensure that the battery always runs in a suitable operating point. Nonlinear effects near the upper and lower bounds of the battery energy are not modeled.

Capacity model The capacity is a variable which changes in long terms. Because it is a key value for the lifetime of the battery it is needed in the economical and the ecological models. From this point of view it cannot be treated as a parameter but as an output of the battery model. The number of charging cycles C specified for a battery is specified by the producer. After C charging cycles the initial capacity cap_0 has dropped down to cap_C. We assume a linear function for decreasing the capacity shown in figure 33

Figure 34 shows a simple model to calculate the capacity cap depending from the parameters (cap_0, cap_C, C). One charging cycle is a complete throughput of an amount of energy which is equal to the capacity. [3] Since the absolute value from the energy flow

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[3] This capacity should be the current capacity rather than the nominal capacity which is specified by the manufacturer. We make a very simple linear approximation by using cap_0 rather than cap.

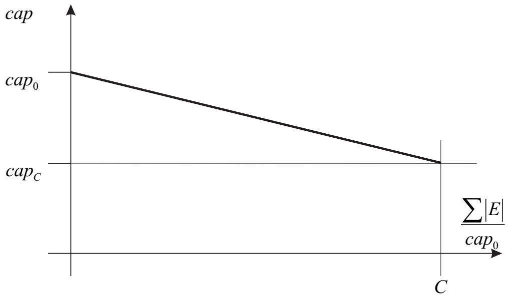

Figure 33: Linear function for decreasing battery capacity.

is taken the energy is integrated twice, once while charging and once while discharging. This effect is justified by the factor 2 in the denominator of the gradient d.

$$d = \frac{cap_0 - cap_C}{2 \cdot C} \tag{13}$$

$$cap = cap_0 - d \cdot \sum \frac{|e|}{cap_0} \tag{14}$$

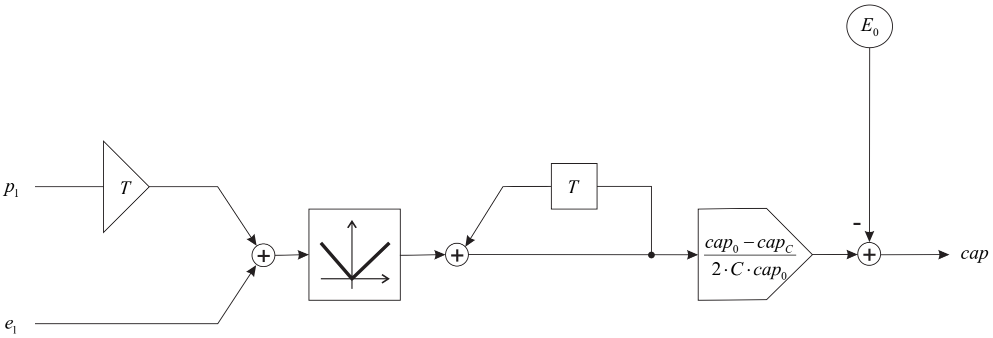

Figure 34: Simple model for the battery capacity.

4.1.2 Bus model

Sliced battery model A bus can contain a single monolithic battery or an array of battery elements with small capacities. We name the second case *sliced battery* concept which has a lot of advantages. If the power electronics could manage that each battery is treated individually in order to optimize the charging and discharging current, to avoid total discharge and minimize possible memory effects. Furthermore during the swapping process the vehicle voltage never drops to zero.

Since a bus with a single battery is a special case of a bus with a sliced battery we assume a sliced battery with n elements including the special case $n = 1$. The swapping process can been seen similarly to the charging process with discrete energy levels and fixed processing time which does not depend on the state of charge of an element.

Basic load The *basic load* of the bus battery is independent of any drive and includes the power for air conditioning, heating and headlights. The basic load is a power p_b

and the needed energy e_b is the time integral of p_b. Since the basic load is mostly constant a multiplication with the operation time t_{opr} is sufficient: $e_b = p_b \cdot t_{opr}$.

Drive load and energy recovery Any bus needs an amount of energy that is in a crude approximation proportional to the properties of the road. The road surface, the length of the road, the speed-dependent wind resistance and the gradient of the road seem to be the main factors. However, the physical reasons for the energy requirements are not important. Any road can be used in two directions. An uphill grade in one direction becomes a downhill grade in the opposite direction. For this reason we define a transportation network over paths and we assign an energy demand e to each *path*.

The bus model defines in a a crude approximation a factor η for a specific bus type. This factor reflects mainly the tire specific rolling resistance and the drag coefficient. The energy consumption on any specific path with a specific bus can be expressed by the product $\Delta e = \eta \cdot e$.

Since the energy demand Δe is negative the bus consumes energy from its batteries [4] . During downhill drives the potential energy reduces the value for Δe but the lower bound is zero. Figure 35 shows an example for a mix of uphill an downhill paths.

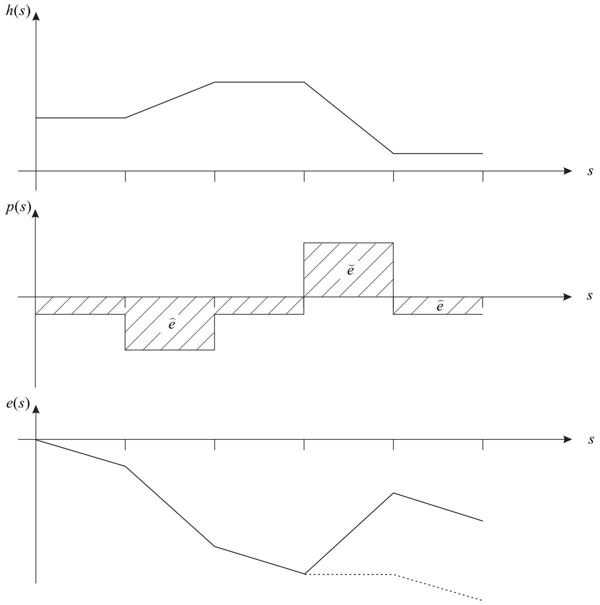

Figure 35: $h(s)$ shows an altitude profile over the path length s, $p(s)$ the corresponding power flow which is negative during the downhill part and $e(s)$ the resulting energy. The solid graph shows $e(s)$ with 100% efficiency energy regeneration. The dashed graph shows $e(s)$ if no energy is regenerated.

Buses with energy recovery reduce the energy demand and e could become positive. Since the efficiency of energy recovery is much lower than 100% we introduce a second path specific factor $\check{\eta}$ and we express the sign of e by $\hat{e}$ for negative values and by $\check{e}$ for positive values. Finally for a bus with energy recovery we get

$$\Delta e = \hat{\eta} \cdot \hat{e} + \check{\eta} \cdot \check{e} p_b \cdot t_{opr} \tag{15}$$

The so far introduced mode consumes energy from the power grid and is called Grid to Vehicle (G2V). The next step to improve the overall efficiency is an energy

---

[4] If *line charging*(see section 4.3.2) is active the energy can be delivered from an external source.

recovery with a feedback to the power grid. This operation mode is called Vehicle to Grid (V2G) and it is not further considered.

4.1.3 Transportation network

The *transportation network* interconnects all bus stops. Mathematically it can be seen as a directed graph $N = (stop, path)$ with the set *stop* of bus stops and the set *path* of connecting paths. Each path consists of one or more road sections, junctions, traffic lights and other traffic specific details which are out of scope for these models.

For all paths we need to know the required energy $\hat{e}$ and if the net is served by buses with energy recovery the offered energy $\breve{e}$ must be known. [5] For further processing we express both energies in vectors $\hat{\boldsymbol{e}}$ and $\breve{\boldsymbol{e}}$ respectively over the paths.

4.2 Operational Models

4.2.1 Modes of operation

We distinguish between *cyclic* and *non cyclic* operation. Cyclic operation is typical for urban areas where the buses drive in loops with mostly fixed time slots. This is convenient for modeling and for evaluation. Section 4.3 deals with cyclic operation.

Non cyclic operation is typical for rural bus lines and has to be treated in another way than cyclic operations.

4.2.2 Principles of energy transfer

Beside the technical principles of energy transfer like pantographs, conductive and inductive charging the energy transfer must be seen more abstract for a mathematical model which should be used for available and future techniques. We distinguish between

- *Charging on lines*: While the bus is driving on the road, the energy for the base load and for the drive load is taken from an external source and the battery is charged.
- *Charging on points*: The bus is charged while it is idle at a bus stop or at the bus depot.
- *Swapping*: Battery swapping is an operation at bus stops or at the bus depot. One or more batteries are exchanged with fully charged batteries.

4.2.3 Timetables and bus schedules

While a timetable is the view of the passenger, the bus schedule is the primary view of the bus company. The timetable specifies the departure times of the buses at the bus stops. Arrival times are not specified in time tables but they are important to know if a point charging at bus stops should be taken into account.

A *bus schedule* is a sequence of bus stops which are to be served by a specific bus. For the bus driver and for our model it is important when the drive to the next stop starts.

4.2.4 A numerical example

It seems to be helpful to have a small model for discussing all the modeled aspects of the problem. So we introduce such a model consisting of only 4 bus stops connected by 5 roads.

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[5] In systems without any energy recovery we omit the accents and use e for the required energy.

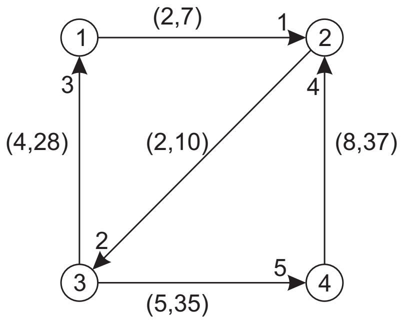

Figure 36: Example for a small transportation network with four bus stops and five paths. $(t, \hat{e}) = (\text{travel time}, \text{energy consumption})$

The structure of this network can be described by its incidence matrix $\boldsymbol{I}$:

$$\boldsymbol{I} = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \tag{16}$$

For some calculations it is useful to know only at which bus stops the paths end. This is the positive part of $\boldsymbol{I}$ which can calculated as

$$\boldsymbol{I}' = \frac{abs(\boldsymbol{I}) + \boldsymbol{I}}{2} \tag{17}$$

$$\boldsymbol{I}' = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{18}$$

The travel time t_d [6] , the stop time t_s [7] at the bus stop and the energy consumption $\hat{e}$ are expressed by vectors over the edges of the transportation network:

$$\boldsymbol{t_d} = \begin{pmatrix} 2 \\ 2 \\ 4 \\ 8 \\ 5 \end{pmatrix} \qquad \boldsymbol{t_s} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad \boldsymbol{\hat{e}} = \begin{pmatrix} 7 \\ 10 \\ 28 \\ 37 \\ 35 \end{pmatrix} \tag{19}$$

4.3 Models for Cyclic Operation

Cyclic operation is typical for urban operation and this operation mode leads to a very simple model if some assumptions hold:

- All buses start their first cycle with sufficiently charged batteries. The charge must give the freedom to locate the point of charging within the cycle independently from the State of Charge (SOC) at the beginning of the cycle.
- The overall cycle time t_c is the least common multiple of the cycle times of all buses. All further calculations are based on t_c.

---

[6] d for drive time

[7] s for stop time

For a cyclic operation the bus schedule can be expressed by a row vector $\boldsymbol{s}$ for every bus. These vectors build the schedule matrix $\boldsymbol{S}$ where $s_{i,j}$ is the frequency of movement of the bus i on the path j in the time interval t_c.

Consider the numerical example from page 29. The schedule matrix $\boldsymbol{S}$ for 3 buses which drive in circles on the introduced network could look like:

$$\boldsymbol{S} = \begin{pmatrix} 5 & 5 & 5 & 0 & 0 \\ 0 & 3 & 0 & 3 & 3 \\ 2 & 4 & 2 & 2 & 2 \end{pmatrix} \tag{20}$$

The bus number 1 drives 5 times the cycle over paths 1, 2 and 3. Bus number 2 drives 3 times the cycle over paths 2, 4 and 5. Bus number 3 drives 2 times an eight-shaped cycle over all paths in which path number 2 is used twice in each cycle.

For a fleet of buses without energy recovery the available energy e for each bus after a finished cycle can be calculated:

$$\boldsymbol{e} = \boldsymbol{e_0} + \hat{\eta} \cdot \boldsymbol{S} \cdot \boldsymbol{\hat{e}} - \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \cdot p_b \cdot t_c \qquad \boldsymbol{e}_0 = \begin{pmatrix} E_{01} \\ \vdots \\ E_{0m} \end{pmatrix} \tag{21}$$

This calculation can be applied iterative over a given number of cycles.

If the buses have different efficiencies $\hat{\eta}_i$ we assign an individual efficiency η_i to each bus i. This can be done for the basic power p_{b_i} as well:

$$\boldsymbol{e} = \boldsymbol{e_0} + \begin{pmatrix} \hat{\eta}_1 & 0 & \cdots & 0 \\ 0 & \hat{\eta}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots \\ 0 & 0 & \cdots & \hat{\eta}_n \end{pmatrix} \cdot \boldsymbol{S} \cdot \boldsymbol{\hat{e}} - \begin{pmatrix} p_{b1} \\ p_{b2} \\ \vdots \\ p_{bn} \end{pmatrix} \cdot t_c \tag{22}$$

$$\boldsymbol{e} = \boldsymbol{e_0} + diag(\boldsymbol{\hat{\eta}}) \cdot \boldsymbol{S} \cdot \boldsymbol{\hat{e}} - \boldsymbol{p_b} \cdot t_c \tag{23}$$

The schedule matrix $\boldsymbol{S}$ shows how often a path is used during a cycle and the matrix $\boldsymbol{S}_s$ shows how often a bus stop is used during a cycle.

$$\boldsymbol{S}_s = \boldsymbol{S} \cdot \boldsymbol{I}'^T \tag{24}$$

$$\boldsymbol{S}_s = \begin{pmatrix} 5 & 5 & 5 & 0 & 0 \\ 0 & 3 & 0 & 3 & 3 \\ 2 & 4 & 2 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{25}$$

$$\boldsymbol{S}_s = \begin{pmatrix} 5 & 5 & 5 & 0 \\ 0 & 3 & 3 & 3 \\ 2 & 4 & 4 & 2 \end{pmatrix} \tag{26}$$

The total operation time $\boldsymbol{t}_{opr}$ is the time while a bus is waiting at a bus stop or it is driving.

$$\boldsymbol{t}_{opr} = \boldsymbol{S} \cdot \boldsymbol{t}_d + \boldsymbol{S}_s \cdot \boldsymbol{t}_s \tag{27}$$

$$\boldsymbol{t}_{opr} = \begin{pmatrix} 5 & 5 & 5 & 0 & 0 \\ 0 & 3 & 0 & 3 & 3 \\ 2 & 4 & 2 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 4 \\ 8 \\ 5 \end{pmatrix} + \begin{pmatrix} 5 & 5 & 5 & 0 \\ 0 & 3 & 3 & 3 \\ 2 & 4 & 4 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} \tag{28}$$

$$\boldsymbol{t}_{opr} = \begin{pmatrix} 40 \\ 45 \\ 46 \end{pmatrix} + \begin{pmatrix} 20 \\ 9 \\ 14 \end{pmatrix} = \begin{pmatrix} 60 \\ 54 \\ 60 \end{pmatrix} \tag{29}$$

The cycle time must be at least as long as the longest travel time:

$$t_c \geq \boldsymbol{S} \cdot \boldsymbol{t}_d + \boldsymbol{S}_s \cdot \boldsymbol{t}_s \tag{30}$$

The start of the next cycle is determined by the timetable and the cycle time and can be derived from the timetable. For a given cycle time t_c we define an idle time t_{idle}:

$$\boldsymbol{t}_{idle} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \cdot t_c - \boldsymbol{S} \cdot \boldsymbol{t}_d + \boldsymbol{S}_s \cdot \boldsymbol{t}_s \tag{31}$$

The energy $\boldsymbol{e}$ for the buses can be calculated by

$$\boldsymbol{e} = \boldsymbol{e_0} + \boldsymbol{S} \cdot \boldsymbol{\hat{e}} + \boldsymbol{p_b} \cdot t_c \tag{32}$$

This is the calculation for the first cycle and $\boldsymbol{e}$ shows the remaining energy for all buses after the completion of the first cycle. A complete cycle is to run before access to the energy vector $\boldsymbol{e}$ is possible and it is a precondition that during a cycle all boundaries (see section 4.3.2 are kept. Equation (32) can be applied recursively where i is the number of the cycle

$$\boldsymbol{e}(0) = \boldsymbol{e_0} \tag{33}$$

$$\boldsymbol{e}(i+1) = \boldsymbol{e}(i) + \boldsymbol{S} \cdot \boldsymbol{\hat{e}} + \boldsymbol{p_b} \cdot t_c \tag{34}$$

4.3.1 Boundaries

In all above given notations the boundaries for the energy of the battery are not taken into account. In all cases the energy of the battery has to be kept in the proper range $\boldsymbol{r} \leq \boldsymbol{e} \leq \boldsymbol{c}$ with the lower bound r_i and the upper bound c_i for the bus i. Usually the upper bound is the capacity of the battery which is unique for any battery or bus respectively.

4.3.2 Charging on lines

Charging on lines includes all techniques which allow charging while driving on a path. It can be modeled by an amount of energy which a path offers or by a power which can be taken by a bus for charging. Since the introduced models are based on mean values both views can be interchanged by each other. In this section we describe an energy based solution.

Any path which is full or only partly equipped for charging offers an amount of energy which can be used by any bus using this path. The offered energy can be taken partly in order to keep the energy in the proper boundaries (see section 4.3.1). The energy $\check{e}$ which is taken by the bus must be split in a part for driving and a part for charging if the battery efficiency should be considered. Otherwise we easily use

$$\boldsymbol{e} = \boldsymbol{e_0} + \boldsymbol{S} \cdot \boldsymbol{\hat{e}} + \boldsymbol{p_b} \cdot t_c + \boldsymbol{S} \cdot \boldsymbol{\check{e}} \tag{35}$$

For a general use we need the recursive form for $\boldsymbol{e}$

$$\boldsymbol{e}(0) = \boldsymbol{e_0} \tag{36}$$

$$\boldsymbol{e}(i+1) = \boldsymbol{e}(i) + \boldsymbol{S} \cdot \boldsymbol{\hat{e}} + \boldsymbol{p_b} \cdot t_c + \boldsymbol{S} \cdot \boldsymbol{\check{e}} \tag{37}$$

where the boundaries must be kept.

4.3.3 Charging on points

Charging on points is equal to charging at bus stops but it shows more the mathematical view of the model. It is only applicable if the stop time is predictable. Because of some non deterministic effects the travel time between the bus stops is non deterministic as well. To take enough energy we need to know a lower bound for the stop time for all bus stops. Otherwise buses could be run into an forced stop with an unacceptable delay.

The stop time vector $\boldsymbol{t}_s$ (see equation (27))contains for all bus stops the time which can be used for charging. The bus stop power vector $\boldsymbol{p}_s$ contains for all bus stops the power which can be offered. For non charging points it is zero.

$$\boldsymbol{e} = \boldsymbol{e_0} + \boldsymbol{S} \cdot \boldsymbol{\hat{e}} + \boldsymbol{p_b} \cdot t_c + \boldsymbol{S}_s \cdot diag(\boldsymbol{p}_s) \cdot \boldsymbol{t}_s \tag{38}$$

For a general use we need the recursive form for $\boldsymbol{e}$

$$\boldsymbol{e}(0) = \boldsymbol{e_0} \tag{39}$$

$$\boldsymbol{e}(i+1) = \boldsymbol{e}(i) + \boldsymbol{S} \cdot \boldsymbol{\hat{e}} + \boldsymbol{p_b} \cdot t_c + \boldsymbol{S}_s \cdot diag(\boldsymbol{p}_s) \cdot \boldsymbol{t}_s \tag{40}$$

where the boundaries must be kept.

4.3.4 Battery switching

Battery swapping is to be done at bus stops or at the bus depot. It is a point operation similar to section 4.3.3 but the energy transfer is modeled not in terms of power and charging time but in discrete values of energy. We assume that all batteries are of the same capacity e_b and batteries which are put in a bus are completely charged. The swapping stations offer a limited amount of batteries which are specified in the vector $\boldsymbol{n}_s$.

$$\boldsymbol{e} = \boldsymbol{e_0} + \boldsymbol{S} \cdot \boldsymbol{\hat{e}} + \boldsymbol{p_b} \cdot t_c + \boldsymbol{S}_s \cdot diag(e_b) \cdot \boldsymbol{n}_s \tag{41}$$

For a general use we need the recursive form for $\boldsymbol{e}$

$$\boldsymbol{e}(0) = \boldsymbol{e_0} \tag{42}$$

$$\boldsymbol{e}(i+1) = \boldsymbol{e}(i) + \boldsymbol{S} \cdot \boldsymbol{\hat{e}} + \boldsymbol{p_b} \cdot t_c + \boldsymbol{S}_s \cdot diag(e_b) \cdot \boldsymbol{n}_s \tag{43}$$

4.4 Time Domain Models

Not all bus lines operate cyclic and some run a long cycle time where it cannot assured that the remaining battery energy is always more than the required reserve res. In these cases the simple accumulative schedule matrix is not sufficient. A sequence of paths or a sequence of timestamps is sufficient and the energy in a bus must only be enough to reach the next hop. Then we can check the boundaries very fine-granular.

Assume the network from figure 36 on page 30. This network should be served by 3 buses all starting from bus stop number 2.

$$\boldsymbol{S}_0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{44}$$

For each bus i the route is to describe by a boolean matrix $\boldsymbol{P}_i$. In accordance to the example in section 4.3 we show these matrices. [8]

$$\boldsymbol{P}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ & & \vdots & & \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \boldsymbol{P}_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ & & \vdots & & \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \boldsymbol{P}_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ & & \vdots & & \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{45}$$

If we need to know at any time which bus is where, we replace the boolean values by start times. All busses start at $t = 0s$ at bus stop 2.

$$\boldsymbol{S}_0 = \begin{pmatrix} \infty & 0 & \infty & \infty \\ \infty & 0 & \infty & \infty \\ \infty & 0 & \infty & \infty \end{pmatrix} \tag{46}$$

All start times for the next hops can be determined easily and the result is

$$\boldsymbol{P}_1 = \begin{pmatrix} 0 & \mathbf{0} & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ & & \vdots & & \\ 0 & 32 & 0 & 0 & 0 \\ 0 & 0 & 34 & 0 & 0 \\ 38 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \boldsymbol{P}_2 = \begin{pmatrix} 0 & \mathbf{0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 7 & 0 \\ & & \vdots & & \\ 0 & 50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 52 \\ 0 & 0 & 0 & 57 & 0 \end{pmatrix} \quad \boldsymbol{P}_3 = \cdots \tag{47}$$

Now we have all the data to keep track of the energy and the charging and swapping can be done similar to the cyclic operation (see section 4.3).

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[8] Although this example is one for cyclic operation the notation can be used for non cyclic operation as well.

5 Ecological model

The ecological model shows the difference to conventional operated bus lines. We consider following aspects:

- The emissions of exhaust gases during operation. Here we have to take into account that the production of electrical energy has an influence to the environment as well even if only renewable energy is used.
- The overall efficiency reduces the advantages for the environment. This effects the energy distribution over the power grid and the limited efficiency of the batteries.
- The ecological footprint for production and scrapping or recycling of the batteries.

From the view of the CACTUS project we only have an influence to the treatment of the batteries to increase their lifetime. The above introduced linear models are not sufficient to express the battery type dependent characteristics. Finally we have to use detailed data from the manufacturer of the batteries. These data should be used in simulation models to estimate the lifetime of the batteries.

The production and scrapping or recycling processes are very individual and therefor we need specific data for the batteries which will be used.

The calculation of the energy demand Δe for a single electrically driven bus is shown in equation (15) where energy recovery is included. The effect on the environment $env_{electric}$ consists of the production of the electricity p and the effects from production b_p and recycling b_r of the batteries. The number of required batteries depends on the number of charging cycles as shown in section 4.1.1 on page 4.1.1. If buses make use of energy recovery, the energy demand decreases. This advantage does not have any influence on the required number of charging cycles C.

$$env_{electric} = (\hat{\eta} \cdot \hat{e} + p_b \cdot t_{opr}) \left(p_{electric} + \left\lceil \frac{C}{C_{max}} \right\rceil (b_p + b_r) \right) \quad + \quad \check{\eta} \cdot \check{e} \cdot p_{electric} \tag{48}$$

$p_{electric}$ summarizes all environmental effects of production, distributing and usage of electrical power.

The above used term $\hat{e}$ of energy demand depends implicit on the length l of the roads to drive. For diesel buses l must be given explicitly. Finally we get for diesel buses the environmental effect env_{diesel}.

$$env_{diesel} = l \cdot \eta_{diesel} \cdot p_{diesel} \tag{49}$$

η_{diesel} is the efficiency of the bus including all influences such as rolling friction, aerodynamic drag and engine efficiency. p_{diesel} summarizes all environmental effects of production, distributing and usage of diesel.

6 Economical Model

6.1 The costs of public transport

The costs of public transport can be divided into the following:

- own costs - the costs incurred in public transport by transport organizers and transport operators,
- costs of transport infrastructure - the costs of infrastructure construction and maintenance which is not covered directly by user fees and costs incurred by state and local authorities,
- external costs - environmental costs and costs of traffic accidents, not covered by user fees,
- cost of time - the cost of time wasted by passengers while travelling.

6.2 Own costs of public transport

The economic capital that must expend and account for a given period of activity of the **transport organizers and transport operators (in Poland, such as KZK GOP and PKM Sosnowiec)**. The structure of these costs is distinguished as following:

- by type of cost: depreciation, fuel, power, tires, wages and deductions from wages, basic materials and other costs,
- by the formation: propellants, oil and lubricants, tires, other materials and objects impermanent, energy, depreciation, repairs, supplies and repairs, wages, deductions from wages, uniforms, business trips, other direct costs, departmental costs, overhead costs and selling expenses.

As carrier of costs shall be in particular:

- **vehicle-kilometer** - ride of vehicle at a distance of one kilometer,
- **vehicle-hour** - ride of vehicle in one hour,
- **vehicle**,
- **route**.

Division costs depend on various carriers:

- costs depend on the number of vehicle-kilometers,
- costs depend on the number of **vehicle-hour**,
- costs depend on the number of rolling stock (number of vehicles),
- costs depend on length of service routes.

The costs depend on the number of vehicle-kilometers:

- propellant, oils and greases,
- electricity for traction purposes,
- tires,

- operational repair and overhaul of rolling stock,
- depreciation of rolling stock.

The costs depend on the number of **vehicle-hour**:

- wages, social security and uniform density of vehicles,
- traffic materials except of fuel, oils, lubricants and tires,
- departmental costs of traffic,
- overhead costs.

The costs depend on the number of rolling stock (number of vehicles):

- depreciation of rolling stock,
- technical inspection.

The costs depend on length of service routes:

- depreciation of network,
- technical inspection of network.

Factors affecting the unit own costs one vehicle-kilometer - These costs are usually determined in practice:

- **the level of costs that depend on:**
  - number of vehicle-kilometer,
  - number of vehicle-hour,
  - number of vehicles (number of rolling stock),
  - length of service routes,
- **service speed on lines** - the number of kilometers traveled per unit of time,
- **the use of rolling stock:**
  - in each day - vehicle-day of operational work,
  - in each hour - vehicle-hour of operational work,
- **the number of vehicles per 1 km of the route**.

Total costs

$$KC_i = PE_{kmi} \cdot k_{kmi} + PE_{hi} \cdot k_{hi} + L_i \cdot k_{li} + W_i \cdot k_{wi}[PLN, EUR] \tag{50}$$

where:

i - index of solution,
PE_{kmi} - operational work [veh-km]
k_{kmii} - unit variable costs of 1 veh-km of operational work [veh-km] in [EUR/1 veh-km]
PE_{hi} - operational work [veh-hours]
k_{hi} - unit fixed costs of 1 veh-hour of operational work [veh-hour] in [EUR/1 veh-hour]
L_i - total length of bus-routes [km]
k_{li} - unit fixed costs of 1 km of bus-routes in [EUR/1 km of routes]
W_i - number of buses
k_{wi} - unit fixed costs of 1 bus [EUR/1 bus]

Total Variable Costs (veh-km dependent)

$$K_{kmi} = K_{mpi} + K_{ogi} + K_{rti} + K_{api} [PLN, EUR] \tag{51}$$

where:

$K_{mpi}[PLN, EUR]$ - costs of: fuel oils, technical lubricant, electric energy for traction (battery),
$K_{ogi}[PLN, EUR]$ costs of tires,
$K_{rti}[PLN, EUR]$ - costs of: repairs, renovations of rolling stock and infrastructure
$K_{api}[PLN, EUR]$ - costs of depreciation (amortization) of rolling stock.

Total Fixed Costs (veh-hour dependent)

$$K_{hi} = K_{vri} + K_{mri} + K_{wri} + K_{ozi} [PLN, EUR] \tag{52}$$

where:

$K_{vri}[PLN, EUR]$ - costs of: wages, social security contributions, uniforms of drivers,
$K_{mri}[PLN, EUR]$ - costs of materials for operational work without fuel and tires,
$K_{wri}[PLN, EUR]$ - departamental costs,
$K_{ozi}[PLN, EUR]$ overhead costs.

Assumption: Total Fixed Cost for comparing variants may be equal

Total Fixed Costs (1 km dependent)

$$K_{li} = K_{aii} + K_{pii} [PLN, EUR] \tag{53}$$

where:

$K_{aii}[PLN, EUR]$ - amortization of infrastructure and equipment for battery charging of e-BUS,
$K_{pii}[PLN, EUR]$ - inspections of infrastructure and equipment.

Total Fixed Costs (1 bus dependent)

$$K_{wi} = K_{awi} + K_{pwi} [PLN, EUR] \tag{54}$$

where:

$K_{awi}[PLN, EUR]$ - amortization of buses,
$K_{pwi}[PLN, EUR]$ - inspections of buses.

All components of costs of operator (PKM Sosnowiec) are as follows:

- the depreciation value of fixed assets,
- the depreciation value of fixed assets purchased after 1995(buses),
- the depreciation value of new infrastructure which were realized after 1995 (except buses),

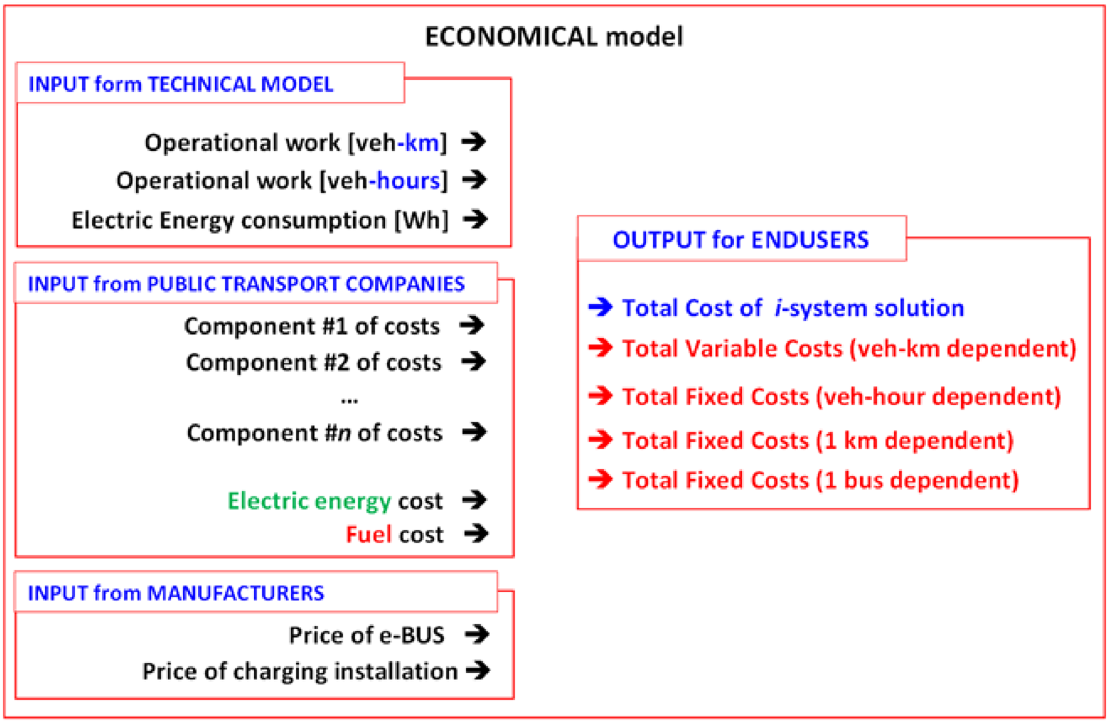

Figure 37: Input-output structure of economical model

- the depreciation value of intangible fixed assets,
- the depreciation value of right of perpetual usufruct of land,
- the depreciation value of bus traction drive departmental costs,
- the depreciation value of workshop usufruct costs,
- the consumption of fuel value,
- the value of used oils and greases,
- the value of used tires,
- the value of other materials which were used (spare parts, lotions, oils, fluids, tools),
- the value of vehicle technical inspection costs,

- the value of office materials, magazines and books,
- the value of workshop electrical energy,
- the value of bus traction drive electrical energy,
- the value of water and liquid wastes used in workshop,
- the value of water and liquid wastes used in bus traction drive
- the value of transport services,
- the value of overhaul (buses),
- the value of repair services and buildings,
- the value of other repair services,
- the value of security, and cleaning of lodging and buses,
- the value of law, IT, bank, tax consulting services,
- The value of connectivity services (telephone, post),
- the value of other external services,
- the value of overhead costs,
- the value of emolument,
- the value of long-service bonus and supplemental (payroll) fund,
- the value of supervisory board emolument,
- the value of mandate contract emolument,
- the value of social contributions of emoluments,
- the value of social contributions of emoluments of mandate contracts,

- the value of allowances to Company Social Benefits Fund,
- the value of expenses connected with Industrial Safety,
- the value of training,
- the value of contribution to additional pension,
- the value of real estate tax,
- the value of vehicle tax,
- the value of stamp-duty,
- the value of PFRON,
- the value of environmental fee,
- the value of notarial and legal costs,
- the value of charge for right of perpetual usufruct of land,
- the value of VAT (structure),
- the value of marketing costs,
- the value of entertainment costs,
- the value of business trips costs,
- the value of VAT (which is not a cost of tax deductible expenses),
- the value of insurance,
- the value of prime costs,
- the value of costs which are not a cost of tax deductible expenses,
- the value of calculation of costs,

- the price of purchase,
- the purchase and installation of charging station costs,
- the value of depreciation of installation of charging station.

6.3 Economical model as a part of general CACTUS model

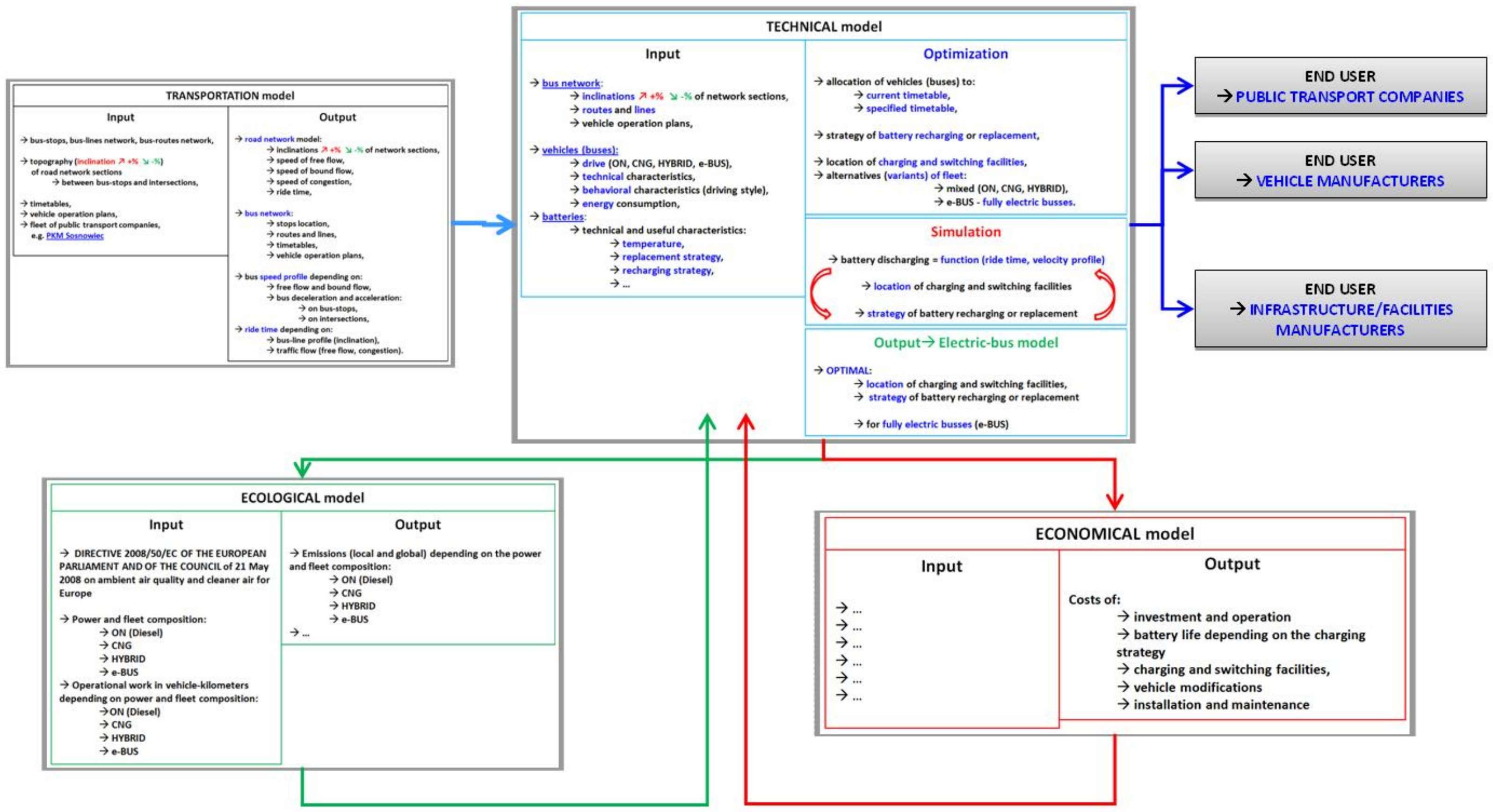

Figure 38: Structure of general CACTUS model and economical model as evaluation tool

List of Figures

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